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ATTENUATION OF BEARING TRANSMITTED NOISE
Volume 1
March 1964

performed in conjunction with

subcontractor
Mechanical Technology Incorporated

in fulfillment of

Contract No. NOBS-86914
Bureau of Ships
Department of Navy
U. S. of America

Westinghouse Electric Corporation
Lester, Pennsylvania

WESTINGHOUSE ELECTRIC CORPORATION

LESTER, PENNSYLVANIA

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Volume 1

Spring and Damping Coefficients For The Tilting-Pad Journal Bearing

by

Jorgan Lund

Mechanical Technology Incorporated

Preface

This report is the first of three volumes which are the product of an analytical and experimental investigation into the effect of a hydraulically supported, tilting-pad, journal bearing on the attenuation of noise originating from rotor unbalance.

The study is a part of a larger program to reduce structure-borne noise originating in high-speed rotating equipment onboard Navy vessels. The introduction of additional flexibility at the journal bearings was proposed to reduce noise transmission, and in the case under study the flexibility is provided by hydraulic pistons and accumulators. In order to evaluate and optimize the additional flexibility, this is a comprehensive investigation of the dynamics of a rotor-bearing system and how the system behavior is affected by the dynamic properties of both the hydrodynamic oil film and the flexible supports.

The second volume of this study will be concerned with the force transmission of (1) two-mass rotor bearing system and (2) uniform rotor bearing system. The third volume will include (1) general rotor analysis and (2) experimental correlation.

Mechanical Technology Incorporated was primarily responsible for the analytical portion of t he investigation while Westinghouse Electric Corp. designed and conducted the experimental test.

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ABSTRACT

A method for calculating spring and damping coefficients for the tilting pad journal bearing is presented. The analysis includes the effect of pad inertia. Numerical results are given in form of design curves for the centrally pivoted 4 shoe, 5 shoe, 6 shoe and 12 shoe journal bearing. A comparison with test results is included.

INTRODUCTION

This report is part of a study of the force transmitted from a rotating shaft to its bearing pedestals where the force is generated by the presence of an unbalance in the rotor. The shaft is supported in fluid film bearings and since the fluid film possesses both flexibility and damping the resulting rotor motion, and therefore the transmitted force, are directly influenced by the properties of the bearing film. Thus it is necessary to determine the bearing stiffness and damping, expressed in terms of spring and damping coefficients, in order to calculate the transmitted force. The bearing spring and damping coefficients depend on the operating conditions and the bearing configuration and have been determined for several bearing types in a preceding investigation, see Ref. 2. In connection with the present continuation of the investigation it has become necessary to examine yet another bearing type, namely the tilting pad bearing.

References 1 and 2 show that although proper bearing design procedures can effect a reduction in the transmitted force, bearing instability imposes an upper limit on the attainable force attenuation. Hence, to further improve the attenuation other methods must be employed. Two possibilities seem open: a) the use of an inherently stable bearing type such as the tilting pad bearing, or b) by vibration isolation of the bearing housing. In order to achieve a substantial force attenuation the latter possibility must be chosen. However, this suggests a very soft bearing support which in turn implies a potential risk for bearing instability. Hence, from both points of view the tilting pad journal bearing offers advantages. For these reasons an analysis of this bearing has been undertaken and it is the purpose of the present report to give the results of the investigation in form of design curves for the spring and damping coefficients. A second report will deal with the force attenuation due to the flexible bearing support.

RESULTS

Numerical calculations have been performed for four bearing geometries: the 4-shoe, the 5-shoe, the 6-shoe, and the 12-shoe tilting pad journal bearing. All four configurations have centrally pivoted pads. The results are given in form of graphs, Figs. 1 to 19, where the spring and damping coefficients are given as functions of the bearing Sommerfeld number. The following factors are investigated:

a) bearing length-to-diameter ratio

b) the direction of the static load

c) the inertia of the shoes

d) the preload of the bearing (for vertical rotors)

The results are given in dimensionless form:

dimensionless spring coefficient: $\frac{CK_{xx}}{W}$, $\frac{CK_{yy}}{W}$

dimensionless damping coefficient: $\frac{C\omega C_{xx}}{W}$, $\frac{C\omega C_{yy}}{W}$

Sommerfeld Number: $S = \frac{\mu NDL}{W} \left(\frac{R}{C}\right)^2$

where:

C	-	Pad radius of curvature minus journal radius, inch
W	-	Bearing reaction, lbs.
D	-	Journal diameter, inch
R	-	Journal radius, inch
L	-	Bearing length, inch
μ	-	Lubricant viscosity, lbs.sec/in.^{2}
N	-	Rotational speed, RPS
ω	-	$2\pi N$, angular speed, rad/sec.
K_{xx}, K_{yy}	-	Spring coefficients, lbs/in.
C_{xx}, C_{yy}	-	Damping coefficients, lbs.sec/in.

Figures 1 to 12 are intended as design curves for horizontal rotors. They cover the three most commonly used bearing geometries for three values of the L/D-ratio. The first 9 graphs have the static load direction between the bottom pads whereas for the next 3 graphs the load vector passes through the pivot of the bottom pad. Since the bearing geometries are symmetric about the vertical load line and shoe inertia is neglected the cross-coupling spring and damp coefficients vanish. Furthermore, for the 4-shoe bearing there is also symmetry around a horizontal line such that coefficients in the horizontal and the vertical direction are identical, i.e. $K_{xx} = K_{yy}$ and $C_{xx} = C_{yy}$. This does not hold for the 5-shoe and the 6-shoe bearing.

On each graph is shown a curve labeled "Critical Mass". It gives the value of the pad inertia necessary to incur resonance of the pad motion, resonance being defined by a 90 degree phase angle between journal and pad motion (or in other words, at resonance the phase angle between a zero-inertia pad and a pad with a "critical mass" is 90 degrees). Hence, for a particular bearing design the value of the dimensionless "pad mass" can be calculated:

$$\text{dimensionless mass} = \frac{CWM}{\left(\mu DL\left(\frac{R}{C}\right)^2\right)^2}$$

where:

M	=	$\frac{I}{R_p^2}$ equivalent pad mass, $\frac{\text{lbs.sec}^2}{\text{in.}}$
I	=	Transverse mass moment of inertia of pad, lbs.in.sec^2
R_p	=	Pad radius, inch.
W	=	Bearing reaction, lbs.
D	=	Journal diameter, inch
R	=	Journal radius, inch
L	=	Bearing length, inch
C	=	Pad clearance, inch
μ	=	Lubricant viscosity, lbs.sec/in.^{2}

Entering the appropriate graph with this value the corresponding "critical" Sommerfeld number, and, therefore, the resonant speed, can be determined.

Figures 13 to 16 illustrate the effect of pad inertia on the spring and damping coefficients for a particular bearing geometry, namely the 4-shoe bearing, L/D = .75, with the load passing between the two bottom pads. The calculations are performed for six values of the dimensionless pad mass. Since the pads are given inertia the cross-coupling terms no longer vanish as for Figs. 1 to 12.

Figures 17 to 19 apply to a vertical rotor with zero bearing eccentricity ratio. In order to have stiffness the bearing must be preloaded. The preload is defined by:

$$\text{Preload} = (1 - C'/C)$$

where:

C = difference between radius of curvature of pad and journal radius, inch.

C' = difference between radius of pivot point circle and journal radius, inch.

Hence, $C'/C = 1$ corresponds to no preload whereas $C'/C = 0$ means that the journal touches the pad.

Since the bearing reaction W is zero for a vertical rotor the coefficients must be made dimensionless in a form different from the horizontal rotor results. The following form is chosen:

vertical rotor

$$\text{dimensionless spring coefficient} = \frac{K_{xx}}{\mu N L \left(\frac{R}{C}\right)^3}$$

$$\text{dimensionless damping coefficient} = \frac{\omega C_{xx}}{\mu N L \left(\frac{R}{C}\right)^3}$$

$$\text{dimensionless critical mass} = \frac{M_{crit}\, N}{\mu L \left(\frac{R}{C}\right)^3}$$

where the symbols are defined above. Note, that for a vertical rotor $K_{xx} = K_{yy}$ and $C_{xx} = C_{yy}$.

All the given results are based on partial arc fluid film force derivative computed by finite difference calculations on a computer using program PN0091, Appendix C. Fluid film rupture is included. Each partial arc (in total 10) is calculated for 8 eccentricity ratios: $\epsilon = .01$, .1, .2, .35, .5, .65, .8 and .95. A summary of the derivatives is given in Table 1.

DISCUSSION OF RESULTS

A comparison between calculated and measured results is shown in Figs. 20 and 21. The test results are taken from Ref. 4. Even if Ref. 4 gives experimental data for values of the Sommerfeld number ranging from 0 to 16, Ref. 5 indicates that the actual measurements are limited to a range from .15 to 1.2 and that outside this range extrapolation has been used. Based on these considerations the agreement seems reasonably good within the normal range of bearing operation. It should also be noted that the test measurements are obtained with vibration amplitudes of up to 50 percent of the minimum film thickness whereas the theoretical calculations assume very small amplitudes.

The most unusual aspect of the theoretical results is the sudden loss of bearing stiffness when approaching pad resonance. However, the effect may be less drastic than shown. Since at resonance the "critical mass" is zero the effect of pad inertia cannot be neglected. Then Figs. 13 to 16 show that the cross-coupling terms become important. Hence, the overall coefficient may not be zero (see Eq. (10), Ref. 1). Pad resonance occurs in general at high Sommerfeld numbers where the eccentricity ratio is small and thus, the out-of-phase between pad and journal motion may not be too serious. In addition the fluid film damping is large.

Finally, it should be noted that the fluid film force calculations include film rupture. When the bearing is flooded with oil the film can sustain pressures below the ambient and rupture does not take place. Instead the film may cavitate. Therefore, if the present results are applied to the calculation of a flooded bearing there may be minor errors at low Sommerfeld numbers.

NUMERICAL EXAMPLE

Let a bearing have a diameter D = 6 inch and length L = 3 inch, i. e., L/D = .5. The radial pad clearance is C = .0045 inch and the pad center coincides with the bearing center so that there is no preload. The speed is 3600 RPM, i.e., N = 60 RPS, and the corresponding oil viscosity is 14 centipoise, i.e., $\mu = 2.0 \cdot 10^{-6}$ lbs.sec/in^2. With a bearing reaction W = 4,000 lbs. the Sommerfeld number becomes S = .244. When the load is between the pads and pad inertia is neglected the dynamic coefficients for the 4-shoe bearing are obtained from Fig. 7.

$$\frac{CK_{xx}}{W} = 1.15 \qquad K_{xx} = 1.02 \cdot 10^6 \text{ lbs/inch}$$

$$\frac{C\omega C_{xx}}{W} = 5.4 \qquad C_{xx} = 12{,}700 \text{ lbs.sec/inch}$$

The dimensionless critical mass is $5.25 \cdot 10^{-3}$ which gives a critical pad mass moment of inertia I = .69 lbs. in.sec^2. This corresponds approximately to a shoe weight of 180 lbs. which is much larger than the actual weight. Hence, there is no danger of resonance. For a 6-shoe bearing the results are obtained from Fig. 2 as:

$$K_{xx} = 4.3 \cdot 10^6 \text{ lbs/inch} \qquad K_{yy} = 1.4 \cdot 10^6 \text{ lbs/inch}$$

$$C_{xx} = 10{,}400 \text{ lbs.sec/inch} \qquad C_{yy} = 3{,}400 \text{ lbs.sec/inch}$$

critical pad moment of inertia = 18.5 lbs.in.sec^2
corresponding to a shoe weight of 12,500 lbs.

DISCUSSION OF ANALYSIS

The theoretical equation governing the fluid film behavior is the well-known Reynolds equation. Due to its complexity it is normally solved numerically on a computer and the resulting force is a non-linear function of the eccentricity ratio and the journal center velocity. For an exact solution Reynolds equation should be solved simultaneously with the equation of motion for the rotor, leading to quite involved calculations. For practical purposes it is convenient instead to assume that the rotor amplitude is sufficiently small to allow replacing the fluid film forces by their gradients around the steady state operating eccentricity. Thus, the forces become proportional to the vibratory amplitude and velocity, the coefficients for proportionality being denoted spring and damping coefficients. Introducing an x-y-coordinate system the fluid film forces can be written as:

$$F_x = -K_{xx}x - C_{xx}\dot{x} - K_{xy}y - C_{xy}\dot{y}$$
$$F_y = -K_{yx}x - C_{yx}\dot{x} - K_{yy}y - C_{yy}\dot{y}$$

such that there are 4 spring coefficients and 4 damping coefficients.

For a fixed pad the spring and damping coefficients can be computed from the gradients of the fluid film force as shown in the Analysis Section or in Refs. 1, 2, 3 and 6. The gradients are obtained by numerical differentiation of computer results.

The analysis of the tilting pad bearing assumes the spring and damping coefficients for the fixed pad to be known. Thus, for an arbitrary rotor motion it becomes possible to establish the equation of motion for the pad itself, including the inertia of the pad. Coupling this equation with the above equations for the fluid film force yields the spring and damping coefficients for the tilting pad. A summation over all pads results in the combined spring and damping coefficients for the complete tilting pad journal bearing.

ANALYSIS

Steady State Equilbrium

Referring to Fig. 22, let the center of the tilting pad bearing be O_B. The bearing is made up of a number of tilting pads, the arbitrary pad having the pivot point P located in an angle ψ from the vertical load line. The pad is free to tilt around the pivot point which for convenience is assumed to be located on the surface of the pad. The steady state position of the journal center is O_J which is the origin of two fixed coordinate systems: the x-y-system (x-axis vertical downward, y-axis horizontal) and the ξ-η-system (the ξ-axis passing through the pivot point). The location of O_J with respect to the bearing center is given by the eccentricity $O_B O_J = e_o = C\varepsilon_o$ and the attitude angle φ_o. The steady state position of the pad center is designated O_n such that the journal center eccentricity with respect to the pad is determined by $O_n O_J = e = C\varepsilon$. The corresponding attitude angle is φ. The journal radius is R, the radius of the pad is $O_n P = R + C$ and the radius of the circle passing through all pivot points with center in O_B is $O_B P = R + C'$.

The point O_{no} is the pad center with no tilting. Hence, $O_{no} O_n$ is a circular arc, or for small motions, a line perpendicular to $O_{no}P$. Projecting O_J on $O_{no}P$ yields:

(1) $$\varepsilon \cos\varphi = 1 - \frac{C'}{C} - \varepsilon_o \cos(\psi - \varphi_o)$$

This equation contains three unknowns, namely ε, φ and φ_o since ε_o is the independent variable. The second equation derives from the requirement that the force on the pad passes through the pivot point which establishes a relationship between ε and φ (i.e. the journal center locus with respect to the pad). The third relationship is the requirement that the total horizontal force component (in the y-direction), summed over all pads, is zero:

(2) $$\sum_{\substack{\text{all}\\ \text{pads}}} F \sin\psi = 0$$

Eq. (2) is usually used to determine φ_0 by trial-and-error as follows: for a particular case $\varepsilon_0, \frac{C'}{C}$ and ψ are known in Eq.(1). For several assumed values of φ_0 calculate $\varepsilon\cos\varphi$ for each pad, determine the pad forces F from available pad data and plot $\sum F\sin\psi$ as a function of φ_0. The zero-point determines the desired value of φ_0.

This procedure is at best tedious. It becomes very complicated if the pivot point is not in the center of the pad in which case the pad force F can be a multivalued function of $\varepsilon\cos\varphi$. However, when the pivot points are located symmetrically with respect to the vertical load line through the bearing center O_B then $\varphi_0 = 0$. A further simplification arises when the pivot point is in the center of the pad since the pad force F then is uniquely determined by $\varepsilon\cos\varphi$. These two conditions apply to all the numerical calculations in the present report but is not a necessary assumption for the analysis to be valid. The analysis requires only that ε and φ are known for each pad. How these values have been arrived at is immaterial in so far as the analysis is concerned.

Fixed Pad Coefficients

In order to determine the spring and damping coefficients for the complete tilting pad bearing it is necessary to know the forces and their derivatives for each pad as if the pad was fixed. Under steady state conditions the journal center has the eccentricity ratio ε and the attitude angle φ with respect to the pad center. The fluid film pad force has the components F_ξ and F_η and under steady state conditions the resultant force passes through the pivot point, i.e. $F_\xi = -F$ and $F_\eta = 0$, see Fig. 22. Thus F denotes the load on the pad. Usually the force is resolved along the radial and the tangential directions with the components F_r and F_t, respectively, see Fig. 22. Hence:

$$\text{(3)} \qquad \begin{Bmatrix} F_\xi \\ F_\eta \end{Bmatrix} = \begin{Bmatrix} -F \\ 0 \end{Bmatrix} = -\begin{Bmatrix} \cos\varphi & \sin\varphi \\ \sin\varphi & -\cos\varphi \end{Bmatrix} \begin{Bmatrix} F_r \\ F_t \end{Bmatrix}$$

For an infinitessimal small motion around the steady state position the dynamic forces become from Eq. (3):

$$(4)\qquad \begin{Bmatrix} dF_\xi \\ dF_\eta \end{Bmatrix} = -\begin{Bmatrix} \cos\varphi & \sin\varphi \\ \sin\varphi & -\cos\varphi \end{Bmatrix}\begin{Bmatrix} dF_r + F_t\, d\varphi \\ dF_t - F_r\, d\varphi \end{Bmatrix}$$

The infinitessimal dynamic motion of the journal center is described by the coordinates (ξ, η):

$$\xi = d(e\cos\varphi) \qquad \eta = d(e\sin\varphi)$$

or

$$(5)\qquad \begin{Bmatrix} de \\ e\,d\varphi \end{Bmatrix} = \begin{Bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{Bmatrix}\begin{Bmatrix} \xi \\ \eta \end{Bmatrix}$$

The velocities transform similiarly:

$$(6)\qquad \begin{Bmatrix} d\dot{e} \\ e\,d\dot{\varphi} \end{Bmatrix} = \begin{Bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{Bmatrix}\begin{Bmatrix} \dot{\xi} \\ \dot{\eta} \end{Bmatrix}$$

The dynamic force components dF_r and dF_t may be expressed in terms of the dynamic amplitudes. From Reynolds equation it can be shown that the fluid film force F can be written (Refs. 1,2):

$$(7)\qquad F = \lambda\omega\left(1-2\frac{\dot{\varphi}}{\omega}\right)\cdot f\left(\varepsilon, \varphi, \left(\frac{\dot{\varepsilon}}{\omega}\right)/\left(1-2\frac{\dot{\varphi}}{\omega}\right)\right)$$

where:

$$(8)\qquad \lambda = \frac{\mu R L}{\pi}\left(\frac{R}{C}\right)^2$$

$$(9)\qquad f = \frac{1}{S_P}$$

$$(10)\qquad S_P = \frac{\mu N D L}{F}\left(\frac{R}{C}\right)^2 \qquad \text{(Pad Sommerfeld Number)}$$

Therefore:

$$(11)\qquad dF = \lambda\omega\left\{\left(1-2\frac{\dot{\varphi}}{\omega}\right)\left[\frac{\partial f}{\partial \varepsilon}d\varepsilon + \frac{\partial f}{\varepsilon\partial\varphi}\varepsilon\, d\varphi + \frac{\partial f}{\partial\left(\frac{\dot{\varepsilon}}{\omega}/\left(1-2\frac{\dot{\varphi}}{\omega}\right)\right)}d\left(\frac{\frac{\dot{\varepsilon}}{\omega}}{1-2\frac{\dot{\varphi}}{\omega}}\right)\right] - \frac{2f}{\omega\varepsilon}\varepsilon\, d\dot{\varphi}\right\}$$

Now:

$$d\left(\frac{\frac{\dot{\varepsilon}}{\omega}}{1-2\frac{\dot{\varphi}}{\omega}}\right) = \frac{1}{1-2\frac{\dot{\varphi}}{\omega}}\, d\left(\frac{\dot{\varepsilon}}{\omega}\right) + \frac{2\frac{\dot{\varepsilon}}{\omega}}{(1-2\frac{\dot{\varphi}}{\omega})^2}\, d\left(\frac{\dot{\varphi}}{\omega}\right) = d\left(\frac{\dot{\varepsilon}}{\omega}\right)$$

because at the equilibrium position $\dot{\varepsilon} = \dot{\varphi} = 0$. Hence, Eq.(11) reduces to:

$$(12) \qquad dF = \frac{1}{c}\lambda\omega\left\{ \frac{\partial f}{\partial \varepsilon}\, de + \frac{\partial f}{\varepsilon\partial\varphi}\, e d\varphi + \frac{1}{\omega}\frac{\partial f}{\partial(\frac{\dot{\varepsilon}}{\omega})}\, d\dot{e} - \frac{1}{\omega}\frac{2f}{\varepsilon}\, e d\dot{\varphi} \right\}$$

Eq.(12) applies to both F_r and F_t. Let the corresponding dimensionless forces be denoted f_r and f_t as defined by Eq. (7). Thus, by substitution of Eq. (12) into Eq. (4):

$$(13) \qquad \begin{Bmatrix} dF_\xi \\ dF_\eta \end{Bmatrix} = -\frac{1}{c}\lambda\omega \begin{Bmatrix} \cos\varphi & \sin\varphi \\ \sin\varphi & -\cos\varphi \end{Bmatrix} \left[\begin{Bmatrix} \frac{\partial f_r}{\partial\varepsilon} & \left(\frac{\partial f_r}{\varepsilon\partial\varphi} + \frac{f_t}{\varepsilon}\right) \\ \frac{\partial f_t}{\partial\varepsilon} & \left(\frac{\partial f_t}{\varepsilon\partial\varphi} - \frac{f_r}{\varepsilon}\right) \end{Bmatrix} \begin{Bmatrix} de \\ e d\varphi \end{Bmatrix} + \frac{1}{\omega} \begin{Bmatrix} \frac{\partial f_r}{\partial(\frac{\dot{\varepsilon}}{\omega})} & -\frac{2f_r}{\varepsilon} \\ \frac{\partial f_t}{\partial(\frac{\dot{\varepsilon}}{\omega})} & -\frac{2f_t}{\varepsilon} \end{Bmatrix} \begin{Bmatrix} d\dot{e} \\ e d\dot{\varphi} \end{Bmatrix} \right]$$

Define the fixed pads spring and damping coefficients by:

$$(14) \qquad \begin{aligned} dF_\xi &= -K_{\xi\xi}\,\xi - C_{\xi\xi}\,\dot{\xi} - K_{\xi\eta}\,\eta - C_{\xi\eta}\,\dot{\eta} \\ dF_\eta &= -K_{\eta\xi}\,\xi - C_{\eta\xi}\,\dot{\xi} - K_{\eta\eta}\,\eta - C_{\eta\eta}\,\dot{\eta} \end{aligned}$$

To determine the 8 coefficients, substitute Eq. (5) and (6) into Eq.(13) and collect the terms in accordance with Eq.(14) to get:

$$(15) \qquad K_{\xi\xi} = \frac{1}{c}\lambda\omega\left[\frac{\partial f_r}{\partial\varepsilon}\cos^2\varphi - \frac{\partial f_t}{\varepsilon\partial\varphi}\sin^2\varphi - \left(\frac{\partial f_r}{\varepsilon\partial\varphi} - \frac{\partial f_t}{\partial\varepsilon}\right)\cos\varphi\sin\varphi - \frac{f_\eta}{\varepsilon}\sin\varphi\right]$$

$$(16) \qquad \omega C_{\xi\xi} = \frac{1}{c}\lambda\omega\left[\frac{\partial f_r}{\partial(\frac{\dot{\varepsilon}}{\omega})}\cos^2\varphi + \frac{\partial f_t}{\partial(\frac{\dot{\varepsilon}}{\omega})}\cos\varphi\sin\varphi - \frac{2f_\xi}{\varepsilon}\sin\varphi\right]$$

$$(17) \qquad K_{\xi\eta} = \frac{1}{c}\lambda\omega\left[\frac{\partial f_r}{\varepsilon\partial\varphi}\cos^2\varphi + \frac{\partial f_t}{\partial\varepsilon}\sin^2\varphi + \left(\frac{\partial f_t}{\varepsilon\partial\varphi} + \frac{\partial f_r}{\partial\varepsilon}\right)\cos\varphi\sin\varphi + \frac{f_\eta}{\varepsilon}\cos\varphi\right]$$

$$(18) \qquad \omega C_{\xi\eta} = \frac{1}{c}\lambda\omega\left[\frac{\partial f_t}{\partial(\frac{\dot{\varepsilon}}{\omega})}\sin^2\varphi + \frac{\partial f_r}{\partial(\frac{\dot{\varepsilon}}{\omega})}\cos\varphi\sin\varphi + \frac{2f_\xi}{\varepsilon}\cos\varphi\right]$$

$$(19) \qquad K_{\eta\xi} = \frac{1}{c}\lambda\omega\left[-\frac{\partial f_t}{\partial\varepsilon}\cos^2\varphi - \frac{\partial f_r}{\varepsilon\partial\varphi}\sin^2\varphi + \left(\frac{\partial f_t}{\varepsilon\partial\varphi} + \frac{\partial f_r}{\partial\varepsilon}\right)\cos\varphi\sin\varphi + \frac{f_\xi}{\varepsilon}\sin\varphi\right]$$

$$(20) \qquad \omega C_{\eta\xi} = \frac{1}{c}\lambda\omega\left[-\frac{\partial f_t}{\partial(\frac{\dot{\varepsilon}}{\omega})}\cos^2\varphi + \frac{\partial f_r}{\partial(\frac{\dot{\varepsilon}}{\omega})}\cos\varphi\sin\varphi - \frac{2f_\eta}{\varepsilon}\sin\varphi\right]$$

$$(21) \quad K_{\eta\eta} = \frac{1}{C}\lambda\omega\left[-\frac{\partial f_t}{\varepsilon\partial\varphi}\cos^2\varphi + \frac{\partial f_r}{\partial\varepsilon}\sin^2\varphi + \left(\frac{\partial f_r}{\varepsilon\partial\varphi} - \frac{\partial f_t}{\partial\varepsilon}\right)\cos\varphi\sin\varphi - \frac{f_\xi}{\varepsilon}\cos\varphi\right]$$

$$(22) \quad \omega C_{\eta\eta} = \frac{1}{C}\lambda\omega\left[\frac{\partial f_r}{\partial(\frac{\dot{\varepsilon}}{\omega})}\sin^2\varphi - \frac{\partial f_t}{\partial(\frac{\dot{\varepsilon}}{\omega})}\cos\varphi\sin\varphi + \frac{2f_\eta}{\varepsilon}\cos\varphi\right]$$

where:

$$(23) \quad f_\xi = \frac{-1}{S_p}$$

$$(24) \quad f_\eta = 0$$

$$(25) \quad \frac{1}{C}\lambda\omega = \frac{1}{C} F S_p$$

and all forces and derivatives are calculed for the given steady state position, defined by ϵ.

<u>Tilting Pad Coefficients</u>

Referring to Fig. 22, 0_n is the steady state position of the pad center. Under dynamic load the pad center oscillates around 0_n with the amplitude η_p such that η_p/R_p represents the dynamic tilting angle of the pad (R_p is the distance from the actual pad pivot to the pad center). The moment on the pad from the fluid film pressure is $-R_p\, dF_\eta$. Therefore, if the mass moment of inertia of the pad is I the equation of motion becomes:

$$I\frac{\ddot{\eta}_p}{R_p} = -R_p\, dF_\eta$$

or

$$(26) \quad \frac{I}{R_p^2}\ddot{\eta}_p = M\ddot{\eta}_p = -dF_\eta$$

where:

$$(27) \quad M = \frac{I}{R_p^2}$$

Thus, under dynamic conditions η should be replaced by ($\eta-\eta_p$) in Eq. (14). In order to eliminate η_p substitute the expression for dF_η into Eq. (26):

(28) $$M\ddot{\eta}_p = K_{\eta\xi}\xi + C_{\eta\xi}\dot{\xi} + K_{\eta\eta}(\eta-\eta_p) + C_{\eta\eta}(\dot{\eta}-\dot{\eta}_p)$$

Let the dynamic motion of the journal center around the steady state position be harmonic:

(29) $$(\xi, \eta)e^{i\omega t}$$

Hence, solve Eq. (28):

(30) $$\eta-\eta_p = -\frac{(K_{\eta\xi}+i\omega C_{\eta\xi})\xi + M\omega^2\eta}{K_{\eta\eta}-M\omega^2+i\omega C_{\eta\eta}} = -\left[(K_{\eta\xi}+i\omega C_{\eta\xi})\xi + M\omega^2\eta\right](p-iq)$$

where:

(31) $$p = \frac{K_{\eta\eta}-M\omega^2}{(K_{\eta\eta}-M\omega^2)^2+(\omega C_{\eta\eta})^2}$$

(32) $$q = \frac{\omega C_{\eta\eta}}{(K_{\eta\eta}-M\omega^2)^2+(\omega C_{\eta\eta})^2}$$

Thus, replacing η by $(\eta-\eta_p)$ Eq. (14) becomes:

(33) $$\begin{aligned} dF_\xi &= -(K'_{\xi\xi}+i\omega C'_{\xi\xi})\xi - (K'_{\xi\eta}+i\omega C'_{\xi\eta})\eta \\ dF_\eta &= -(K'_{\eta\xi}+i\omega C'_{\eta\xi})\xi - (K'_{\eta\eta}+i\omega C'_{\eta\eta})\eta \end{aligned}$$

where $K'_{\xi\xi}$, $C'_{\xi\xi}$ etc, are the spring and damping coefficients for the tilting pad and given by:

(34) $$K'_{\xi\xi} = K_{\xi\xi} - (pK_{\xi\eta}+q\omega C_{\xi\eta})K_{\eta\xi} - (qK_{\xi\eta}-p\omega C_{\xi\eta})\omega C_{\eta\xi}$$

(35) $$\omega C'_{\xi\xi} = \omega C_{\xi\xi} - (pK_{\xi\eta}+q\omega C_{\xi\eta})\omega C_{\eta\xi} + (qK_{\xi\eta}-p\omega C_{\xi\eta})K_{\eta\xi}$$

(36) $$K'_{\xi\eta} = -M\omega^2(pK_{\xi\eta}+q\omega C_{\xi\eta})$$

$$(37)\qquad \omega C'_{\xi\eta} = M\omega^2(qK_{\xi\eta} - p\omega C_{\xi\eta})$$

$$(38)\qquad K'_{\eta\xi} = -M\omega^2(pK_{\eta\xi} + q\omega C_{\eta\xi})$$

$$(39)\qquad \omega C'_{\eta\xi} = M\omega^2(qK_{\eta\xi} - p\omega C_{\eta\xi})$$

$$(40)\qquad K'_{\eta\eta} = -M\omega^2(pK_{\eta\eta} + q\omega C_{\eta\eta}) = -M\omega^2(1 + pM\omega^2)$$

$$(41)\qquad \omega C'_{\eta\eta} = M\omega^2(qK_{\eta\eta} - p\omega C_{\eta\eta}) = (M\omega^2)^2 q$$

If the pad has no inertia, i.e. $M=0$, only $K'_{\xi\xi}$ and $\omega C'_{\xi\xi}$ remain as would be expected.

Bearing Spring and Damping Coefficients

Having determined the spring and damping coefficients for the individual tilting pads it remains to combine them into the overall bearing coefficients. The coordinate system for the bearing is the x-y-system, see Fig. 22, with the coordinate transformation:

$$(42)\qquad \left\{\begin{matrix} \xi \\ \eta \end{matrix}\right\} = -\left\{\begin{matrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{matrix}\right\}\left\{\begin{matrix} x \\ y \end{matrix}\right\}$$

$$(43)\qquad \left\{\begin{matrix} dF_x \\ dF_y \end{matrix}\right\} = -\left\{\begin{matrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{matrix}\right\}\left\{\begin{matrix} dF_\xi \\ dF_\eta \end{matrix}\right\}$$

where dF_x and dF_y are the dynamic forces measured in the x-y-system. The bearing spring and damping coefficients are defined by:

$$(44)\qquad \begin{aligned} dF_x &= -K_{xx}x - C_{xx}\dot{x} - K_{xy}y - C_{xy}\dot{y} \\ dF_y &= -K_{yx}x - C_{yx}\dot{x} - K_{yy}y - C_{yy}\dot{y} \end{aligned}$$

Thus, substituting Eq. (42) and (43) into Eq. (33) and grouping terms in accordance with Eq. (44) yields:

$$(45)\qquad K_{xx} = K'_{\xi\xi}\cos^2\psi + K'_{\eta\eta}\sin^2\psi - (K'_{\xi\eta} + K'_{\eta\xi})\cos\psi\sin\psi$$

$$\omega C_{xx} = \omega C'_{\xi\xi}\cos^2\psi + \omega C'_{\eta\eta}\sin^2\psi - (\omega C'_{\xi\eta} + \omega C'_{\eta\xi})\cos\psi\sin\psi \tag{46}$$

$$K_{xy} = K'_{\xi\eta}\cos^2\psi - K'_{\eta\xi}\sin^2\psi + (K'_{\xi\xi} - K'_{\eta\eta})\cos\psi\sin\psi \tag{47}$$

$$\omega C_{xy} = \omega C'_{\xi\eta}\cos^2\psi - \omega C'_{\eta\xi}\sin^2\psi + (\omega C'_{\xi\xi} - \omega C'_{\eta\eta})\cos\psi\sin\psi \tag{48}$$

$$K_{yx} = K'_{\eta\xi}\cos^2\psi - K'_{\xi\eta}\sin^2\psi + (K'_{\xi\xi} - K'_{\eta\eta})\cos\psi\sin\psi \tag{49}$$

$$\omega C_{yx} = \omega C'_{\eta\xi}\cos^2\psi - \omega C'_{\xi\eta}\sin^2\psi + (\omega C'_{\xi\xi} - \omega C'_{\eta\eta})\cos\psi\sin\psi \tag{50}$$

$$K_{yy} = K'_{\eta\eta}\cos^2\psi + K'_{\xi\xi}\sin^2\psi + (K'_{\xi\eta} + K'_{\eta\xi})\cos\psi\sin\psi \tag{51}$$

$$\omega C_{yy} = \omega C'_{\eta\eta}\cos^2\psi + \omega C'_{\xi\xi}\sin^2\psi + (\omega C'_{\xi\eta} + \omega C'_{\eta\xi})\cos\psi\sin\psi \tag{52}$$

A summation over all the pads making up the bearing gives the bearing spring and damping coefficients. If the pads have no inertia the equations simplify to:

$$\begin{aligned} K_{xx} &= K'_{\xi\xi}\cos^2\psi & \omega C_{xx} &= \omega C'_{\xi\xi}\cos^2\psi \\ K_{xy} = K_{yx} &= K'_{\xi\xi}\cos\psi\sin\psi & \omega C_{xy} = \omega C_{yx} &= \omega C'_{\xi\xi}\cos\psi\sin\psi \\ K_{yy} &= K'_{\xi\xi}\sin^2\psi & \omega C_{yy} &= \omega C'_{\xi\xi}\sin^2\psi \end{aligned} \tag{54}$$

Thus, for symmetry around the x-axis and no pad inertia the cross-coupling terms disappear.

Pad Motion

The pad motion is given by Eq.(30) which can also be written:

$$\eta_p = \frac{(K_{\eta\xi} + i\omega C_{\eta\xi})\xi + (K_{\eta\eta} + i\omega C_{\eta\eta})\eta}{K_{\eta\eta} - M\omega^2 + i\omega C_{\eta\eta}} \tag{55}$$

Let $\eta_o = \eta_p$ for M = 0, i.e. for no pad inertia:

$$\eta_o = \eta + \frac{K_{\eta\xi} + i\omega C_{\eta\xi}}{K_{\eta\eta} + i\omega C_{\eta\eta}}\,\xi \tag{56}$$

Then:

$$(57)\qquad \frac{\eta_P}{\eta_0} = \frac{K_{\eta\eta}+i\omega C_{\eta\eta}}{K_{\eta\eta}-M\omega^2+i\omega C_{\eta\eta}} = 1+\frac{M\omega^2}{K_{\eta\eta}-M\omega^2+i\omega C_{\eta\eta}} = 1+pM\omega^2-iqM\omega^2$$

or:

$$(58)\qquad \left|\frac{\eta_P}{\eta_0}\right| = \sqrt{(1+pM\omega^2)^2+(qM\omega^2)^2}$$

$$(59)\qquad arg(\eta_0)-arg(\eta_P) = \tan^{-1}\left(\frac{qM\omega^2}{1+pM\omega^2}\right) = \tan^{-1}\left(\frac{\omega C_{\eta\eta}\, M\omega^2}{K_{\eta\eta}(K_{\eta\eta}-M\omega^2)+(\omega C_{\eta\eta})^2}\right)$$

Eq. (58) gives the amplitude ratio, i.e. the magnification factor, and Eq. (59) gives the phase angle lag with respect to a inertialess pad. Thus the two equations indicate how well the pad follows the shaft motion. The phase angle becomes 90° when:

$$(60)\qquad M_{crit}\,\omega^2 = \frac{K_{\eta\eta}^2+(\omega C_{\eta\eta})^2}{K_{\eta\eta}}$$

Hence, if the pad mass satisfies Eq. (60) there will be a resonance of the pad motion.

It is convenient to use Eq. (60) to establish a value for M, designated the critical mass. In dimensionless form Eq. (60) may be written:

$$(61)\qquad \frac{CWM_{crit}}{[\mu DL(\frac{R}{C})^2]^2} = \frac{1}{4\pi^2 S}\,\frac{\left(\frac{CK_{\eta\eta}}{W}\right)^2+\left(\frac{C\omega C_{\eta\eta}}{W}\right)^2}{\frac{CK_{\eta\eta}}{W}}$$

where S is the bearing Sommerfeld number.

CONCLUSIONS

1. An analytical method has been established for calculating the spring and damping coefficients of the tilting pad journal bearing. The coefficients can be used directly in computing the critical speed, the response and the transmitted force of a rotor.

2. Numerical results for several bearing configurations have been obtained and are presented in form of design curves. A comparison with test results shows fair agreement.

3. The results indicate a sudden reduction in stiffness when approaching resonance of the pad motion. The implications of this behavior have not been assessed.

RECOMMENDATIONS

1. Since the force attentuation obtainable with a tilting pad journal bearing is not necessarily limited by oil whip, a study should be undertaken on how to optimize the attenuation by a proper choice of bearing dimensions, notably the clearance.

2. The present analysis considers the motion of the shoes around an axial axis, i.e. the "rolling" of the shoes, a more complete investigation should incorporate both "pitch" and "yaw" of the shoe.

3. Although a tilting pad journal bearing is considered "inherently stable" this holds true only in the idealized case where the shoes have no inertia and the pivots are frictionless. Hence, the stability limit of a tilting pad bearing should be studied, especially the stability of the shoe motion.

REFERENCES

1. Lund, J.W. and Sternlicht, B., "Rotor-Bearing Dynamics with Emphasis on Attenuation," Journal of Basic Engineering, Transaction of the ASME, Vol. 84, Series D., No. 4 Dec. 1962.

2. Lund, J.W. and Sternlicht, B., "Bearing Attenuation," Bureau of Ships Report, Contract No. NObs-78930, April 28, 1961.

3. Warner, P.C., "Static and Dynamic Properties of Partial Journal Bearings," Journal of Basic Engineering, Transaction of the ASME, Vol. 85, Series D. No. 2 June 1963.

4. Hagg, A.C. and Sankey, G.O., "Elastic and Damping Properties of Oil-Film Journal Bearings for Application to Unbalance Vibration Calculations," Journal of Applied Mechanics, Vol. 25, Trans. ASME, Vol. 80, 1958, p. 141.

5. Hagg, A.C. and Sankey, G.O. "Some Dynamic Properties of Oil-Film Journal Bearings with Reference to the Unbalance Vibration of Rotors," Journal of Applied Mechanics, Vol. 23, Trans. ASME, Vol. 78, 1956, pp. 302-306.

6. Warner, P.C. and Thoman, R.J., "The Effect of the 150-degree Partial Bearing on Rotor-Unbalance Vibration," Transactions of the ASME, Paper No. 63-LubS-6, presented June 2-5, 1963, Boston.

7. Boyd, J. and Raimondi, A.A., "An Analysis of the Pivoted-Pad Journal Bearing," Mechanical Engineering, Vol. 75, 1953, p.380.

APPENDIX A

COMPUTER PROGRAM PN0078: SPRING AND DAMPING COEFFICIENTS FOR THE TILTING PAD JOURNAL BEARING

APPENDIX A

Computer Program PN0078: Spring and Damping Coefficients for the Tilting Pad Journal Bearing

The program calculates the spring and damping coefficients for the tilting pad journal bearing based on the analysis given in this report. The input to the program consists of the partial arc bearing forces and force derivatives, the position of the pads and the steady state bearing eccentricity ratio. The forces and their derivatives can be obtained from PN0091, Appendix C, and data reduced by PN0131, Appendix B. The output from the latter program serves directly as input to PN0078.

Input Data

The program is written for the IBM 1620 computer, 40K memory storage, with input and output on punched cards.

Card 1 and 2

Descriptive text, Col. 2 to 52

Card 3

7 · (I5)

Word 1 (NP) gives the number of pads. NP $\leq$ 12
Word 2 (NEB) gives the number of bearing eccentricity ratios for which calculations are performed. There is no upper limit on NEB.
Word 3 (NM) gives the number of calculations performed for each bearing eccentricity ratio with different values of the pad inertia. The maximum value of NM is 12. If the pad inertia is neglected, set NM = 1.
Word 4 (NF) controls the meaning of the input items f_r and f_t in the pad data (f_r = radial force component, f_t = tangential force component). If NF = 0, then the input value for f_r is the pad Sommerfeld number and the input value for f_t is the horizontal pad force f_η (usually $f_\eta = 0$). If NF $\neq$ 0, then $(f_r)_{input} = f_r$ and $(f_t)_{input} = f_t$.

<u>Word 5</u> (MC) controls the way in which the pad inertia data is given as input. If MC= -1, each pad will be given its own inertia. If MC = 0, each pad has zero inertia. If MC = +1, all the pads have the same inertia.

<u>Word 6</u> (MS) controls the form in which the pad inertia is made dimensionless in the input. Denote the input value $\bar{M}$. Then:

$$\text{if} \quad MS=-1 \qquad \bar{M} = \frac{1}{S}\frac{MN^2}{W/C}$$

$$\text{if} \quad MS=0 \qquad \bar{M} = \frac{MN^2}{W/C}$$

$$\text{if} \quad MS=+1 \qquad \bar{M} = \frac{1}{S^2}\frac{MN^2}{W/C} = \frac{MWC}{\left[\mu DL\left(\frac{R}{C}\right)^2\right]^2}$$

where:

- C Radial pad clearance, inch
- D Journal diameter, inch
- L Bearing length, inch
- I Mass moment of inertia of pad around longitudinal axis, lbs-in-sec^2
- M $= I/R^2$, equivalent pad mass, $\text{lbs-sec}^2/\text{in.}$
- N Rotational speed, RPS
- R Journal radius, inch
- S $= \frac{\mu NDL}{W}\left(\frac{R}{C}\right)^2$, overall bearing Sommerfeld Number
- W Total bearing reaction, lbs.
- μ Lubricant viscosity, lbs-sec/in^2

In general it is convenient to use the last form of $\bar{M}$ (i.e. for MS = +1) since it is independent of speed.

<u>Word 7</u> (NPR). If NPR = 1, the output includes the calculated values of the pad spring and damping coefficients in addition to the overall bearing coefficients. If NPR = 0, the pad coefficients are not given, only the bearing coefficients.

<u>List of Pad Position Angles</u>

<u>4 · (E15.7)</u>

This input list defines the position of the pads by means of the angle ψ measured from the vertical to the pivot point of the pad (See Fig.22). ψ is measured in degrees. There must be NP-values (word 1, card 3), 4 values per card.

List of Equivalent Pad Masses if MC = +1
4(E15.7)

This input list is used when it is desired to study the effect of pad inertia in the calculations and all pads have identical inertia. This form of the input list can only be supplied when MC=+1 (word 5, card 3). The pad inertia is given in form of the dimensionless equivalent pad mass $\bar{M}$. The definition of $\bar{M}$ depends on the value of MS (word 6, card 3). The input list must contain as many values of $\bar{M}$ as given by NM (word 3, card 3), 4 values per card. Thus it is possible to obtain the dynamic bearing coefficients as functions of the pad inertia.

List of Pad Forces and Derivatives

The spring and damping coefficients are calculated from the pad forces and their derivatives. These quantities can be computed by means of PN0091, Appendix C, and data-reduced by program PN0131, Appendix B. The output cards from PN0131 can be used directly to make up the present input list. The input list consists of a number of sets of complete bearing data, identified by a bearing eccentricity ratio. There are as many sets as given by NEB (word 2, card 3). Each set comprises the forces and their derivatives for each pad. To illustrate:

1st ϵ_o

Card 1 (3(E15.7)) ε_0 φ_0 C'/C

1st Pad
- Card 2a (4(E15.7)) ε φ f_r f_t
- Card 3a (4(E15.7)) $\partial f_r/\partial\varepsilon$ $\partial f_t/\partial\varepsilon$ $\partial f_r/\varepsilon\partial\varphi$ $\partial f_t/\varepsilon\partial\varphi$
- Card 4a (4(E15.7)) $\partial f_r/\partial(\frac{\dot{\varepsilon}}{\omega})$ $\partial f_t/\partial(\frac{\dot{\varepsilon}}{\omega})$

2nd Pad
- Card 2b (4(E15.7)) ε φ f_r f_t

last pad

Card 4x (4(E15.7))

2nd ϵ_o

Card 1 (3(E15.7))

1st Pad

Card 2 (4(E15.7))

where ε_o = bearing eccentricity ratio, see Fig. 22
φ_o = bearing attitude angle, degrees, see Fig. 22
C'/C = ratio of pivot circle clearance to pad clearance
ε = pad eccentricity ratio, see Fig.22
φ = pad attitude angle
f_r = dimensionless radial force component
f_t = dimensionless tangential force component
ω = angular speed of journal, rad/sec

"Card 1" above is actually not used by the program in the calculations. It serves only the purpose of identification. Furthermore, it should be noted that ϵ cannot be zero since the calculations include f_r/ϵ and f_t/ϵ.

<u>List of Equivalent Pad Masses if MC = -1</u>
<u>4(E15.7)</u>

This input is used when the pads have different inertia. This form of the input list can only be supplied when MC=-1 (word 5, card 3). The pad inertia is given in the form of the dimensionless equivalent pad mass $\bar{M}$ as defined through the value of MS (word 6, Card 3). An input list must be given for each bearing eccentricity ratio, i.e. following "Card 4X" for each ϵ_o above. The list is made up of NM - sets (word 3, card 3) and each set contains NP values of $\bar{M}$ (word 1, card 3). To illustrate:

1st ϵ_o

last pad: Card 4x | $\partial f_r/\partial(\frac{\dot{\varepsilon}}{\omega})$ | $\partial f_t/\partial(\frac{\dot{\varepsilon}}{\omega})$ } List of Pad forces and Derivatives

$\bar{M}_{pad\ 1}$ $\bar{M}_{pad\ 2}$ $\bar{M}_{pad\ 3}$ $\bar{M}_{pad\ 4}$
$\bar{M}_{pad\ 5}$ – – – – $\bar{M}_{pad\ NP}$
$\bar{M}_{pad\ 1}$ $\bar{M}_{pad\ 2}$ – – – – –
– – – – – $\bar{M}_{pad\ NP}$
} List of Equivalent Pad Masses for MC = -1. NM sets

2nd ϵ_o

Card 1 ε_o φ_o C'/C } List of Pad Forces and Derivatives (continued)

Output Data

The output first repeats the input data for identification and checking purposes. Then follow the results for each bearing eccentricity ratio. The text identifying the numerical values is explained below except where the meaning is obvious:

SOMMERF.NO. - When part of pad results: $S_n = \frac{\mu NDL}{F}\left(\frac{R}{C}\right)^2$ (F=total force on pad)

When part of bearing results: $S = \frac{\mu NDL}{W}\left(\frac{R}{C}\right)^2$ (W=total force on bearing)

F - T = f_η

F X I = $-1/S_p$, where S_p is the pad Sommerfeld Number.

K11,K12,K21,K22 = $K_{\xi\xi}$, $K_{\xi\eta}$, $K_{\eta\xi}$, $K_{\eta\eta}$ i.e. the 4 spring coefficients for the fixed pad.

WC11,WC12,WC21,WC22 = $\omega C_{\xi\xi}$, $\omega C_{\xi\eta}$, $\omega C_{\eta\xi}$, $\omega C_{\eta\eta}$ i.e.the 4 damping coefficients for the fixed pad.

The 8 coefficients are dimensionless in the form:

$$K_{\xi\xi} = \frac{C}{F}\bar{K}_{\xi\xi} \qquad \omega C_{\xi\xi} = \frac{C}{F}\omega\bar{C}_{\xi\xi}$$

where F is the total force on the pad. Thus K11 is calculated as:

$$K11 = S_p\left[\frac{\partial f_r}{\partial \varepsilon}\cos^2\varphi - \frac{\partial f_t}{\varepsilon\partial\varphi}\sin^2\varphi - \left(\frac{\partial f_r}{\varepsilon\partial\varphi} - \frac{\partial f_t}{\partial\varepsilon}\right)\cos\varphi\sin\varphi - \frac{f_\eta}{\varepsilon}\right]$$

and similarly for the other coefficients, see Eq. (15).

CRIT.MASS $= \frac{CFM_{crit}}{\left(\mu DL\left(\frac{R}{C}\right)^2\right)}$ where part of pad results.

$= \frac{CWM_{crit}}{\left(\mu DL\left(\frac{R}{C}\right)^2\right)}$ where part of bearing results.

KD11,KD12,KD21,KD22 = $K'_{\xi\xi}$, $K'_{\xi\eta}$, $K'_{\eta\xi}$, $K'_{\eta\eta}$ i.e. the spring coefficients for the tilting pad.

WCD11,WCD12,WCD21,WCD22 = $\omega C'_{\xi\xi}$, $\omega C'_{\xi\eta}$, $\omega C'_{\eta\xi}$, $\omega C'_{\eta\eta}$ i.e. the damping coefficients for the tilting pad.

The coefficients are dimensionless in the form:

$$K'_{\xi\xi} = \frac{C}{F}\bar{K}'_{\xi\xi} \qquad \text{etc.}$$

KXX,KXY,KYX,KYY = $K_{xx}, K_{xy}, K_{yx}, K_{yy}$, i.e. the spring coefficients for the complete bearing.

WCXX,WCXY,WCYX,WCYY = $\omega C_{xx}, \omega C_{xy}, \omega C_{yx}, \omega C_{yy}$, i.e.the damping coefficients for the complete bearing.

The coefficients are dimensionless in the form: $K_{xx} = \frac{C}{W} \overline{K_{xx}}$, $\omega C_{xx} = \frac{C}{W} \omega \overline{C_{xx}}$ etc. where W is the total force on the bearing. Thus for instance K_{xx} is calculated internally in the program directly on the basis of the input values for the pad forces and their derivatives and the final result is multiplied by the bearing Sommerfeld Number S.

DIMENSIONLESS PAD MASS - is the pad mass as defined through MS (word 6, card 3, input).

AMPLITUDE = $|\eta_p/\eta_o|$, i.e. the ratio between the actual pad angular amplitude and the angular amplitude of a massless pad, see Eq. (58). Hence, the ratio may be thought of as a magnification factor.

PHASE ANG = $\arg(\eta_o) - \arg(\eta_p)$, i.e. the phase angle in degrees between the angular motion of a massless pad and the angular motion of the actual pad.

$$\left.\begin{array}{ll} \text{CALC. MASS} & = \dfrac{1}{S}\dfrac{M\omega^2}{W/C} \\ \text{MASS 1} & = \dfrac{1}{S}\dfrac{MN^2}{W/C} \\ \text{MASS 2} & = \dfrac{MN^2}{W/C} \\ \text{MASS 3} & = \dfrac{1}{S^2}\dfrac{MN^2}{W/C} = \dfrac{MWC}{(\mu DL(\frac{R}{C})2)^2} \end{array}\right\}$$

Four different forms of the dimenionless pad inertia. For details, see explanation in "Input Data", Card 3, Word 6.

CRIT. SP.RATIO = $M_{crit}/M = \omega_{crit}/\omega$, i.e. the ratio between the resonant pad frequency ω_{crit} and the operating angular journal speed for the given Sommerfeld Number.

INPUT FORM FOR

PN0078: Spring and Damping Coefficients for the Tilting Pad Journal Bearing.

IBM 1620 Computer, FORTRAN I

Card 1: Text, Col. 2-52

____

Card 2: Text, Col. 2-52

____

Card 3: 7 · (I5)

______ 1. NP. Number of pads. NP $\leq$ 12

______ 2. NEB. Number of bearing eccentricity ratios

______ 3. NM. Number of calculations including pad inertia per bearing eccentricity ratio. NM $\leq$ 12.

______ 4. NF. If NF = 0, input value of f_r is pad Sommerfeld number and input value of f_t is horizontal pad force f_η. If NF = 1, $f_r = f_r$ and $f_t = f_t$.

______ 5. MC. If MC = -1: each pad is given its own inertia

If MC = 0: all pads have zero inertia

If MC = +1: all pads have same inertia

______ 6. MS If MS = -1: dimensionless pad inertia = $\frac{1}{S}\frac{MN^2}{W/C}$

If MS = 0: dimensionless pad inertia = $\frac{MN^2}{W/C}$

If MS = +1: dimensionless pad inertia = $\frac{1}{S^2}\frac{MN^2}{W/C} = \frac{MWC}{(\mu DL(\frac{R}{C})2)^2}$

______ 6. NPR If NPR = 0: output does not include results for individual pads

If NPR = 1: output includes results for individual pads

List of Pivot Point Angles: 4(E15.7)

Give the position angle, degrees, of the pivot point for each pad, in total NP — values, 4 values per card.

<u>List of Dimensionless Pad Inertia, MC = +1</u>: 4 · (E15.7)

Use this list only when MC = +1. Give the dimensionless pad inertia (same for all pads) according to value of MS, in total NM - values, 4 values per card:

List of Bearing and Pad Data: 4 (E15.7)

Give NEB-sets of cards. If MC ≠ -1, each set contains (1+3 · NP) cards. If MC = -1, each set contains $(1 + 3NP + MN \cdot \left\{\frac{NP}{4}\right\})$ cards:

ε_o φ_o c'/c

ε φ f_r f_t

$\frac{\partial f_r}{\partial \varepsilon}$ $\frac{\partial f_t}{\partial \varepsilon}$ $\frac{\partial f_r}{\varepsilon \partial \varphi}$ $\frac{\partial f_t}{\varepsilon \partial \varphi}$ 1st Pad

$\partial f_r/\partial(\frac{\dot\varepsilon}{\omega})$ $\partial f_t/\partial(\frac{\dot\varepsilon}{\omega})$

ε φ f_r f_t

$\frac{\partial f_r}{\partial \varepsilon}$ $\frac{\partial f_t}{\partial \varepsilon}$ $\frac{\partial f_r}{\varepsilon \partial \varphi}$ $\frac{\partial f_t}{\varepsilon \partial \varphi}$ 2nd Pad

$\partial f_r/\partial(\frac{\dot\varepsilon}{\omega})$ $\partial f_t/\partial(\frac{\dot\varepsilon}{\omega})$

— " —

— " — 3rd Pad

— " —

— " —

— " — 4th Pad

— " —

$\bar{M}_{Pad\,1}$, - -

- - $M_{Pad\,NP}$ Pad inertia if MC=-1

2nd Bearing Eccentricity Ratio

______	______	______		ε_o φ_o c'/c	
______	______	______	______	ε φ f_r f_t	1st pad
______	______	______	______	$\frac{\partial f_r}{\partial \varepsilon}$ $\frac{\partial f_t}{\partial \varepsilon}$ $\frac{\partial f_r}{\varepsilon \partial \varphi}$ $\frac{\partial f_t}{\varepsilon \partial \varphi}$	
______	______			$\partial f_r/\partial(\frac{\dot{\varepsilon}}{\omega})$ $\partial f_t/\partial(\frac{\dot{\varepsilon}}{\omega})$	
______	______	______	______	ε φ f_r f_t	2nd pad
______	______	______	______	$\frac{\partial f_r}{\partial \varepsilon}$ $\frac{\partial f_t}{\partial \varepsilon}$ $\frac{\partial f_r}{\varepsilon \partial \varphi}$ $\frac{\partial f_t}{\varepsilon \partial \varphi}$	
______	______			$\partial f_r/\partial(\frac{\dot{\varepsilon}}{\omega})$ $\partial f_t/\partial(\frac{\dot{\varepsilon}}{\omega})$	
______	______	______	______	— " —	3rd pad
______	______	______	______	— " —	
______	______			— " —	
______	______	______	______	— " —	4th pad
______	______	______	______	— " —	
______	______			— " —	
______	______	______	______	$\bar{M}_{Pad\,1}$, $\bar{M}_{Pad\,2}$, --	Pad inertia if MC=-1
______	______	______	______	--- $\bar{M}_{Pad\,NP}$	
______	______	______	______	$\bar{M}_{Pad\,1}$, $\bar{M}_{Pad\,2}$, --	
______	______	______	______	--- $\bar{M}_{Pad\,NP}$	

```
      DIMENSION PVA(20),CS(20),SN(20),CSQ(20),SNQ(20),CSN(20),P(20)
      DIMENSION B(8),GXX(20),GXY(20),GYX(20),GYY(20),HXX(20),HXY(20)
      DIMENSION HYX(20),HYY(20),CMS(20)
   50 READ 200
      READ 201
      READ 202,NP,NEB,NM,NF,MC,MS,NPR
      PUNCH 203
      PUNCH 200
      PUNCH 201
      PUNCH 204
      PUNCH 205,NP,NEB,NM,NF,MC,MS,NPR
      PUNCH 206
      DO 51 I=1,NP,4
      READ 207,PVA(I),PVA(I+1),PVA(I+2),PVA(I+3)
   51 PUNCH 207,PVA(I),PVA(I+1),PVA(I+2),PVA(I+3)
      IF (MC) 54,54,52
   52 DO 53 I=1,NM,4
   53 READ 207,P(I),P(I+1),P(I+2),P(I+3)
   54 DO 55 I=1,8
   55 B(I)=0.0
      DO 100 I=1,NP
      C2=.017453293*PVA(I)
      C1=COS(C2)
      C2=SIN(C2)
      CS(I)=C1
      SN(I)=C2
      CSQ(I)=C1*C1
      SNQ(I)=C2*C2
  100 CSN(I)=C2*C1
      DO 157 I=1,NEB
      READ 208,ECB,ATB,CRT
      PUNCH 210
      PUNCH 209,ECB,ATB,CRT
      BSN=0.0
      FH=0.0
      DO 120 J=1,NP
      READ 207,ECP,ATP,FR,FT,DFRE,DFTE,DFRA,DFTA,DFRS,DFTS
      C1=.017453293*ATP
      CSP=COS(C1)
      SNP=SIN(C1)
      CSPQ=CSP*CSP
      SNPQ=SNP*SNP
      CS2=SNP*CSP
      IF(NF) 101,104,101
  101 C1=FR
      C2=FT
      FXI=-FR*CSP-FT*SNP
      FT=-FR*SNP+FT*CSP
      FR=SQRT(FR*FR+C2*C2)
      IF(FR)103,102,103
  102 FR=1.0E90
      GO TO 105
  103 FR=1.0/FR
      GO TO 105
  104 FXI=-1.0/FR
  105 BSN=BSN+FXI*CS(J)-FT*SN(J)
      FH=FH-FXI*SN(J)-FT*CS(J)
      PUNCH 211, J
      PUNCH 212
      PUNCH 213,ECP,ATP,FR,FT,DFRE
      PUNCH 214
```

```
      PUNCH 213,DFTE,DFRA,DFTA,DFRS,DFTS
      IF (NF) 106,107,106
  106 PUNCH 215
      PUNCH 213,C1,C2,FXI
  107 C1=(DFRE+DFTA)*CS2
      C2=(DFTE-DFRA)*CS2
      C3=DFRS*CS2
      C4=DFTS*CS2
      IF(ECP) 109,108,109
  108 C5=0.0
      C6=0.0
      GO TO 110
  109 C5=FXI/ECP
      C6=FT/ECP
  110 AXX=DFRE*CSPQ-DFTA*SNPQ+C2-C6*SNP
      AXY=DFRA*CSPQ+DFTE*SNPQ+C1+C6*CSP
      AYX=-DFTE*CSPQ-DFRA*SNPQ+C1+C5*SNP
      AYY=-DFTA*CSPQ+DFRE*SNPQ-C2-C5*CSP
      C5=2.0*C5
      C6=2.0*C6
      BXX=DFRS*CSPQ+C4-C5*SNP
      BXY=DFTS*SNPQ+C3+C5*CSP
      BYX=-DFTS*CSPQ+C3-C6*SNP
      BYY=DFRS*SNPQ-C4+C6*CSP
      IF (AYY) 56,55,56
   55 C10=0.0
      GO TO 57
   56 C10=(AYY+BYY/AYY*BYY)/39.478418
   57 CMS(J)=C10
      IF (NPR) 111,112,111
  111 C1=FR*AXX
      C2=FR*AXY
      C3=FR*AYX
      C4=FR*AYY
      C5=FR*BXX
      C6=FR*BXY
      C7=FR*BYX
      C8=FR*BYY
      C9=C10/FR
      PUNCH 216
      PUNCH 213,C1,C2,C3,C4
      PUNCH 217
      PUNCH 213, C5,C6,C7,C8,C9
  112 IF (MC) 119,113,119
  113 C1=AYY*AYY+BYY*BYY
      IF (C1) 115,114,115
  114 PUNCH 218,J
      GO TO 120
  115 C2=AXY*AYX-BXY*BYX
      C3=AXY*BYX+AYX*BXY
      C4=AXX-(AYY*C2+BYY*C3)/C1
      C5=BXX-(AYY*C3-BYY*C2)/C1
      C1=CSQ(J)
      C2=SNQ(J)
      C3=CSN(J)
      B(1)=B(1)+C4*C1
      B(2)=B(2)+C4*C3
      B(3)=B(2)
      B(4)=B(4;+C4*C2
      B(5)=B(5)+C5*C1
      B(6)=B(6)+C5*C3
```

```
      B(7)=B(6)
      B(8)=B(8)+C5*C2
      IF (NPR) 117,118,117
  117 PUNCH 219,C4,C5
  118 GO TO 120
  119 GXX(J)=AXX
      GXY(J)=AXY
      GYX(J)=AYX
      GYY(J)=AYY
      HXX(J)=BXX
      HXY(J)=BXY
      HYX(J)=BYX
      HYY(J)=BYY
  120 CONTINUE
      BSN=1.0/BSN
      IF (MC) 123,121,123
  121 DO 60 J=1,8
   60 B(J)=BSN*B(J)
      PUNCH 220
      PUNCH 213,B(1),B(2),B(3),B(4),B(5)
      PUNCH 221
      PUNCH 213,B(6),B(7),B(8),BSN,FH
      PUNCH 228
      DO 158 J=1,NP
      C9=CMS(J)/BSN
  158 PUNCH 229, J,C9
      DO 122 J=1,8
  122 B(J)=0.0
      GO TO 157
  123 DO 156 K=1,NM
      IF (MC) 124,126,126
  124 DO 125 J=1,NP,4
  125 READ 207,P(J),P(J+1),P(J+2),P(J+3)
      GO TO 127
  126 PM=P(K)
      PUNCH 223,PM
  127 DO 151 J=1,NP
      IF (MC)128,129,129
  128 PM=P(J)
  129 PZ=39.478418*PM
      IF (MS) 130,131,132
  130 PC=PZ
      P1=PM
      P2=BSN*PM
      P3=PM/BSN
      GO TO 133
  131 PC=PZ/BSN
      P1=PM/BSN
      P2=PM
      P3=P1/BSN
      GO TO 133
  132 PC=PZ*BSN
      P1=PM*BSN
      P2=P1*BSN
      P3=PM
  133 AXX=GXX(J)
      AXY=GXY(J)
      AYX=GYX(J)
      AYY=GYY(J)
      BXX=HXX(J)
      BXY=HXY(J)
```

```
      BYX=HYX(J)
      BYY=HYY(J)
      C3=AYY-PC
      C4=C3*C3+BYY*BYY
      IF (C4) 137,134,137
  134 IF (AYY) 136,135,136
  135 PUNCH 218,J
      GO TO 151
  136 PUNCH 222,J
      GO TO 151
  137 C1=C3/C4
      C2=BYY/C4
      C3=1.0+C1*PC
      C4=C2*PC
      ETP=SQRT(C3*C3+C4*C4)
      IF (C3) 146,138,146
  138 IF (BYY)142,139,142
  139 IF (AYX+BYX) 141,140,141
  140 PUNCH 218,J
      GO TO 151
  141 ETP=0.0
      PHA=180.0
      GO TO 150
  142 IF (C4) 145,143,144
  143 PHA=0.0
      GO TO 150
  144 PHA=90.0
      GO TO 150
  145 PHA=-90.0
      GO TO 150
  146 C4=C4/C3
      PHA=57.295780*ATAN(C4)
      IF (C3) 147,150,150
  147 IF (C4) 148,150,149
  148 PHA=PHA+180.0
      GO TO 150
  149 PHA=PHA-180.0
  150 C5=-PC*C3
      C6=PC*C4
      C7=C1*AXY+C2*BXY
      C8=C1*BXY-C2*AXY
      GR=AXX-C7*AYX+C8*BYX
      GE=BXX-C7*BYX-C8*AYX
      DR=-PC*C7
      DE=-PC*C8
      C3=-PC*(C1*AYX+C2*BYX)
      C4=-PC*(C1*BYX-C2*AYX)
      C8=CMS(J)/BSN
      IF (P3) 161,160,161
  160 C9=1.0E+09
      GO TO 162
  161 C9=C8/P3
  162 PUNCH 211, J
      PUNCH 224
      PUNCH 213,GR,DR,C3,C5,GE
      PUNCH 225
      PUNCH 213,DE,C4,C6,ETP,PHA
      PUNCH 226
      PUNCH 213,PC,P1,P2,P3
      PUNCH 227
      PUNCH 213, C8,C9
```

```
      C8=CSN(J)
      C1=(DR+C3)*C8
      C2=(DE+C4)*C8
      C7=(GR-C5)*C8
      C8=(GE-C6)*C8
      AXX=CSQ(J)
      BXX=SNQ(J)
      B(1)=B(1)+GR*AXX+C5*BXX-C1
      B(2)=B(2)+DR*AXX-C3*BXX+C7
      B(3)=B(3)+C3*AXX-DR*BXX+C7
      B(4)=B(4)+C5*AXX+GR*BXX+C1
      B(5)=B(5)+GE*AXX+C6*BXX-C2
      B(6)=B(6)+DE*AXX-C4*BXX+C8
      B(7)=B(7)+C4*AXX-DE*BXX+C8
      B(8)=B(8)+C6*AXX+GE*BXX+C2
  151 CONTINUE
      IF (BSN) 152,154,152
  152 DO 153 J=1,8
  153 B(J)=BSN*B(J)
  154 PUNCH 220
      PUNCH 213,B(1),B(2),B(3),B(4),B(5)
      PUNCH 221
      PUNCH 213,B(6),B(7),B(8),BSN,FH
      DO 155 J=1,8
  155 B(J)=0.0
  156 CONTINUE
  157 CONTINUE
      STOP
  200 FORMAT(49H0                                                ,3H   )
  201 FORMAT(49H0                                                ,3H   )
  202 FORMAT(I5,I5,I5,I5,I5,I5,I5)
  203 FORMAT(49H1PN0078 SPR-DAMP.COEFF.FOR TILT.PAD JRNL.BRG.1-18,3H-63)
  204 FORMAT(49H0  N.PDS  N.ECC  N.MSS  FORCE  M.INP  M.TPE  PRT)
  205 FORMAT(1XI5,2XI5,2XI5,2XI5,2XI5,2XI5,2XI5)
  206 FORMAT(19H0PIVOT POINT ANGLES)
  207 FORMAT(E15.7,E15.7,E15.7,E15.7)
  208 FORMAT(E15.7,E15.7,E15.7)
  209 FORMAT(9H BRG.ECC=,E14.7,12H    ATT.ANG=,E14.7,9H    CP/C=,E14.7)
  210 FORMAT(2H0 /20H0BEARING CALCULATION)
  211 FORMAT(11H0PAD NUMBER,I3)
  212 FORMAT(4X8HECC.RAT.7X7HATT.ANG5X10HSOMMERF.NO8X3HF-T9X6HDFR/DE)
  213 FORMAT(2X E12.5,2X E12.5,2X E12.5,2X E12.5,2X E12.5)
  214 FORMAT(5X6HDFT/DE8X7HDFR/EDA7X7HDFT/EDA6X8HDFR/DE/W6X8HDFT/DE/W)
  215 FORMAT(7X2HFR12X2HFT12X3HFXI)
  216 FORMAT(7X3HK1111X3HK1211X3HK2111X3HK22)
  217 FORMAT(6X4HWC1110X4HWC1210X4HWC2110X4HWC228X9HCRIT.MASS)
  218 FORMAT(18H0MOTION OF PAD NO.,I3,14H INDETERMINATE)
  219 FORMAT(6H KD11=E12.5,6X6HWCD11=E12.5)
  220 FORMAT(2H0 /1H 6X3HKXX11X3HKXY11X3HKYX11X3HKYY10X4HWCXX)
  221 FORMAT(6X4HWCXY10X4HWCYX10X4HWCYY6X11HSOMMERF.NO.4X10HHORIZ.FRC.)
  222 FORMAT(33H0RESONANCE WITHOUT DAMPING,PAD NO,I3)
  223 FORMAT(24H0DIMENSIONLESS PAD MASS=,E12.5)
  224 FORMAT(6X4HKD1110X4HKD1210X4HKD2110X4HKD2210X5HWCD11)
  225 FORMAT(6X5HWCD129X5HWCD219X5HWCD227X9HAMPLITUDE5X9HPHASE ANG)
  226 FORMAT(4X9HCALC.MASS6X6HMASS 18X6HMASS 28X6HMASS 3)
  227 FORMAT(4X9HCRIT.MASS2X13HCRIT.SP.RATIO)
  228 FORMAT(9X7HPAD NO.2X9HCRIT.MASS)
  229 FORMAT(8XI5,3XE12.5)
      END
```

APPENDIX B

COMPUTER PROGRAM PN0131: INTERPOLATION OF PARTIAL ARC BEARING FORCE DERIVATIVES FOR USE IN TILTING PAD BEARING CALCULATIONS

APPENDIX B

Computer Program PN0131: Interpolation of Partial Arc Bearing Force Derivatives for Use in Tilting Pad Bearing Calculations

In order to calculate the spring and damping coefficients for the tilting pad journal bearing it is necessary to know the force derivatives of the pads making up the complete bearing. Since these derivatives are calculated numerically in a computer by finite difference methods they are usually only available for a limited number of eccentricity ratios. When the pad is incorporated in the tilting pad bearing its eccentricity ratio is determined by the bearing eccentricity ratio (see Eq. (B6)) and it, therefore, becomes necessary to know the force derivatives at other eccentricity ratios than the ones for which calculations have been performed. It is rather costly and impractical to obtain this data by performing additional calculations with the finite difference computer program and instead interpolation of the existing data can be employed. It has been found that to do this manually (by curve plotting) is time consuming. Hence, a computer program has been written for the purpose of performing the interpolation. The computer program accepts as input a list of the force derivatives as computed from the finite difference program (i.e.PN0091, Appendix C). In addition the number of pads and their position must be given. Then the computer program calculates the force derivatives for each pad for a specified number of bearing eccentricity ratios. The output can be used directly as input for the tilting pad bearing program (i.e. PN0078, Appendix A).

Analysis

The interpolation is based on quadratic curve fitting. Let the function to be interpolated be denoted q which is given for a number of discreet eccentricity ratio values by the input. Then:

(B1) $$q = A_i(\varepsilon-\varepsilon_i)^2 + B_i(\varepsilon-\varepsilon_i) + q_i \qquad \underline{\varepsilon_{i-1} \leq \varepsilon \leq \varepsilon_{i+1}}$$

where:

(B2) $$A_i = \frac{\frac{1}{\Delta_2}(q_{i+1}-q_i) - \frac{1}{\Delta_1}(q_i - q_{i-1})}{\Delta_1+\Delta_2}$$

(B3) $$B_i = \frac{\frac{\Delta_1}{\Delta_2}(q_{i+1}-q_i) + \frac{\Delta_2}{\Delta_1}(q_i - q_{i-1})}{\Delta_1+\Delta_2}$$

$$\Delta_1 = \varepsilon_i - \varepsilon_{i-1} \qquad \Delta_2 = \varepsilon_{i+1} - \varepsilon_i$$

Thus, for each ε-interval, except the first and the last, q is determined by two equations. The average value is used:

(B4) $$q = \tfrac{1}{2}\left[A_i(\varepsilon-\varepsilon_i)^2 + B_i(\varepsilon-\varepsilon_i) + q_i + A_{i-1}(\varepsilon-\varepsilon_{i-1})^2 + B_{i-1}(\varepsilon-\varepsilon_{i-1}) + q_{i-1}\right]$$

There are nine functions to be interpolated. Since they all (except the attitude angle) become infinite when $\varepsilon=1$ it is found necessary to normalize the functions before interpolation takes place. The normalization is chosen as follows:

(B5)

$\underline{\varepsilon_1 \leq \varepsilon \leq \varepsilon_{L+1}}$ No normalization

$\underline{\varepsilon_L \leq \varepsilon \leq \varepsilon_n}$ Normalization used: $\begin{cases} f_r, \frac{\partial f_r}{\partial \varepsilon}, \frac{\partial f_r}{\partial(\frac{\dot{\varepsilon}}{\omega})} & \text{is divided by } \frac{\varepsilon^2}{(1-\varepsilon^2)^2} \\ f_t, \frac{\partial f_t}{\partial \varepsilon}, \frac{\partial f_t}{\partial(\frac{\dot{\varepsilon}}{\omega})} & \text{is divided by } \frac{\varepsilon}{(1-\varepsilon)^{3/2}} \\ \frac{\partial f_r}{\varepsilon \partial \varphi}, \frac{\partial f_t}{\varepsilon \partial \varphi} & \text{is divided by } \frac{\varepsilon}{1-\varepsilon^2} \end{cases}$

Here ε_1 and ε_n are the first and last eccentricity ratio, respectively, in the input list. ε_L is specified by the input as described later.

Let the tilting pad bearing have m pads whose pivot points are located the angle ψ from the vertical, measured in the direction of rotation. Then the eccentricity ratio of any pad is determined by:

(B6) $$\varepsilon \cos\varphi = 1 - \frac{C'}{C} - \varepsilon_0 \cos(\psi - \varphi_0)$$

where ε_0 is the bearing eccentricity ratio, φ_0 is the bearing attitude angle, $(1-\frac{C'}{C})$ is the preload, ε is the pad eccentricity ratio and φ is the pad attitude angle. The input specifies $\varepsilon_0, \varphi_0, \frac{C'}{C}$ and ψ and φ is given as a function of ε in the earlier mentioned input list.

Thus, $\varepsilon \cos\varphi$ can be calculated from the input list and by interpolation Eq. (B6) can be solved to find ε for each pad. With ε determined the 8 force parameters can be calculated by interpolation using Eq.(B4) and (B5).

Input Data

The program is written for the IBM 1620 computer, 40K memory storage, with input and output punched on cards.

Card 1
Descriptive text, Column 2 to 52

Card 2
5 · (1XI4)

Word 1 (NEP) gives the number of pad eccentricity ratios for which the force derivatives are provided in the pad data list. NEP $\leq$ 15.

Word 2 (LM) gives the value of L as used in Eq.(B5). If the pad eccentricity ratios in the pad data list are $\varepsilon_1, \varepsilon_2 -- \varepsilon_n$ then the pad data are divided by a scale factor internally in the program for $\varepsilon \geq \varepsilon_L$. Since the scale factor is zero for $\varepsilon = 0$ it has been found necessary to set the scale factor equal to 1 for $\varepsilon \leq \varepsilon_L$. In the determination of L the most critical parameter is $\frac{\partial f_r}{\varepsilon \partial \varphi}$ which is usually negative for $\varepsilon \gtrsim .5$. Hence, it is suggested to give L such a value that $\varepsilon_{L+1} \cong .5$ to $.6$.

Word 3 (NBE) gives the number of tilting pad bearing eccentricity ratios. There is no limit to NBE.

Word 4 (NA) gives the number of pads. NA $\leq$ 20.

Word 5 (INP) If INP=0, more input data follows the present set of input. If INP $\neq$ 0, the present set of input is the last set.

<u>Pad Data List</u>
<u>5 · (1XE13.6)</u>

The pad data is usually obtained from a finite difference solution of Reynolds equation, see Appendix C. The pad forces, force derivatives and attitude angle are calculated for a number of pad eccentricity ratios $\varepsilon_1, \varepsilon_2 --- \varepsilon_n$ and supplied as input to the present computer program. For each eccentricity ratio two cards are given:

First card:	ε	φ	f_r	$\frac{\partial f_r}{\partial \varepsilon}$	$\frac{\partial f_r}{\partial(\dot{\varepsilon}/\omega)}$
Second card:	f_t	$\frac{\partial f_t}{\partial \varepsilon}$	$\frac{\partial f_t}{\partial(\dot{\varepsilon}/\omega)}$	$\frac{\partial f_r}{\varepsilon \partial \varphi}$	$\frac{\partial f_t}{\varepsilon \partial \varphi}$

where:

ε = pad eccentricity ratio

φ = pad attitude angle

$f_r = \frac{F_r}{\mu NDL(R/c)^2}$, dimensionless radial force

$f_t = \frac{F_t}{\mu NDL(R/c)^2}$ dimensionless tangential force

$\dot{\varepsilon}$ = radial journal center velocity, sec^{-1}

Here, φ, f_r and f_t are calculated for the given ε at the steady state position such that the total force passes through the pivot point.

Hence, the pad data list consists of 2 ·(NEP) cards (see word 1, card 2). The list is given in sequence such that $\varepsilon_1 < \varepsilon_2 < --- < \varepsilon_{NEP}$

<u>List of Pivot Point Angles</u>
<u>5 · (1XE13.6)</u>

This input list gives the values of the angle ψ measured from the vertical to the pivot point of each pad in the direction of rotation, see Fig. 22. The angle is measured in degrees. There are (NA)-values in total (word 4, card 2), 5 values per card.

List of Bearing Eccentricity Ratios
3 · (E15.7)

The position of the journal center with respect to the bearing center is defined through the eccentricity ratio ε_0 and the attitude angle φ_0 see Fig. 22. ε_0 is calculated with respect to the pad clearance (pad radius minus journal radius) and φ_0 is in degrees, measured from the vertical to the line of centers in the direction of rotation. In the general case (off-center pivot position, unsymmetric arrangement of the pads) a considerable effort is involved in determining φ_0 but if the pads are centrally pivoted and located symmetrically around the vertical then $\varphi_0 = 0$.

There must be an input card for each bearing eccentricity ratio, in total (NBE) cards. Each card contains three words:

Word 1 gives the bearing eccentricity ratio ε_0
Word 2 gives the bearing attitude angle φ_0 in degrees
Word 3 gives the ratio C'/C where C' is the radius of the pivot point circle minus the journal radius and C is the pad radius minus the journal radius.

Output Data

The output gives first the values of the input data for identification and checking purposes. Then follow 6 lines of text describing the format of the output data and thereafter the numerical values of the output data arranged as follows (FORMAT 4(E15.7):

Card 1, 2 and 3	- blank	
Card 4	- $\varepsilon_0, \varphi_0, C'/C$	
Card 1a	- $\psi, \varepsilon\cos\varphi, \varphi$	1st pad
Card 2a	- $\varepsilon, \varphi, f_r, f_t$	1st pad
Card 3a	- $\partial f_r/\partial\varepsilon, \partial f_t/\partial\varepsilon, \partial f_r/\varepsilon\partial\varphi, \partial f_t/\varepsilon\partial\varphi$	1st pad
Card 4a	- $\partial f_r/\partial(\frac{\dot{\varepsilon}}{\omega}), \partial f_t/\partial(\frac{\dot{\varepsilon}}{\omega})$	1st pad
Card 1b	- $\psi, \varepsilon\cos\varphi, \varphi$	2nd pad
Card 2b	- $\varepsilon, \varphi, f_r, f_t$	2nd pad
⋮	⋮	

Note that two values of the pad attitude angle φ are given. The first value (i.e. on card 1a, 1b etc) is obtained by interpolation of the pad data list whereas the second value (i.e. on card 2a,2b etc) is calculated from $\varphi = \tan^{-1}(f_t/f_r)$. It is the latter value that should be actually used and by comparison with the first φ-value some indications are obtained of the accuracy of the interpolation.

The output cards can be used directly as input to the tilting pad bearing computer program (PN0078, Appendix A) by removing the 3 blank cards in front of each bearing eccentricity ratio calculation (card 1,2 and 3 above) and by removing the first card of each pad data output set (card 1a,1b etc. above).

Returning to Eq. (B6) it can be seen that geometrical considerations limit the range of the bearing eccentricity ratio ε. The program takes care of this automatically as follows. For a given $\varepsilon_o, \varphi_o, c'/c$ and ψ the right hand side of Eq. (B6) may be calculated. Denote the result $(\varepsilon \cos\varphi)_{calc}$. Three cases may arise:

a) If $\varepsilon_1 \cos\varphi_1 \leq (\varepsilon \cos\varphi)_{calc} < 1$, Eq. (B6) is solved by interpolation. Here ε_1 is the first eccentricity ratio value in the pad data list.

b) If $(\varepsilon \cos\varphi)_{calc} < \varepsilon_1 \cos\varphi_1$ the computer writes: "NO LOAD, PVT. ANG=X.XX, E * COS(A)=X.XX" indicating that the particular pad, identified by its pivot angle ψ has a $(\varepsilon \cos\varphi)_{calc}$-value less than the lowest value in the pad data list. Strictly speaking there may be solutions in this case but assuming that ε_1 in general is almost zero it is safest not to allow extrapolation.

c) If $(\varepsilon \cos\varphi)_{calc} \geq 1$ the computer writes: "E=1, PVT. ANG=X.XX, E * COS(A)=X.XX" indicating that for the particular pad the journal is either touching or beyond the pad surface. The calculations proceed with the next pad.

INPUT FORM FOR

PN0131: Interpolation of Partial Arc Bearing Force Derivatives for Use in Tilting Pad Bearing Calculations

IBM 1620 Computer, FORTRAN I

Card 1 (Text Col.2-52)

____

Card 2: 5(1XI4)

______ 1. NEP: Number of pad eccentricity ratios in pad data list. NEP $\leq$ 15

______ 2. LM: $\left.\begin{array}{l}\varepsilon \leq \varepsilon_{LM+1} = \text{pad data not normalized} \\ \varepsilon \geq \varepsilon_{LM} = \text{pad data is normalized}\end{array}\right\}$ see Eq. (B5)

______ 3. NBE: Number of bearing eccentricity ratios

______ 4. NA: Number of pads, NA $\leq$ 20

______ 5. INP: If INP=0: more input follows. If INP$\neq$0, last set of input.

Pad Data List: 5(1XE13.6)

Give 2 cards per pad eccentricity ratio, in total 2 · NEP cards:

____	____	____	____	____	$\varepsilon, \varphi, f_r, \frac{\partial f_r}{\partial \varepsilon}, \frac{\partial f_r}{\partial(\varepsilon\omega)}$
____	____	____	____	____	$f_t, \frac{\partial f_t}{\partial \varepsilon}, \frac{\partial f_t}{\partial(\varepsilon\omega)}, \frac{\partial f_r}{\varepsilon\partial\varphi}, \frac{\partial f_t}{\varepsilon\partial\varphi}$
____	____	____	____	____	— " —
____	____	____	____	____	— " —
____	____	____	____	____	— " —
____	____	____	____	____	— " —
____	____	____	____	____	— " —
____	____	____	____	____	— " —
____	____	____	____	____	— " —
____	____	____	____	____	— " —

<u>Pivot Point Angles</u>: 5(1XE13.6)

Give the position angle(degrees)of the pivot point for each pad, in total NA-values, 5 values per card:

____	____	____	____	____
____	____	____	____	____

<u>Bearing Eccentricity Ratio</u>: 3(E15.7)

Give one card per bearing eccentricity ratio, in total NBE cards:

____	____	____	ε_c , φ_o , c'/c
____	____	____	— " —
____	____	____	— " —
____	____	____	— " —
____	____	____	— " —
____	____	____	— " —

```
      DIMENSION P(11,15),PN(11,15),A(11,15),B(11,15),PA(20),Q(11)
   49 READ 101
      READ 102,NEP,LM,NBE,NA,INP
      PUNCH 100
      PUNCH 101
      PUNCH 104
      PUNCH 103,NEP,LM,NBE,NA,INP
      PUNCH 107
      DO 47 I=1,20
   47 PA(I)=0.0
      DO 48 I=1,11
      Q(I)=0.0
      DO 48 J=1,15
      A(I,J)=0.0
      B(I,J)=0.0
      P(I,J)=0.0
   48 PN(I,J)=0.0
      KM=LM+1
      NM=NEP-1
      DO 55 J=1,NEP
      PUNCH 108,J
      DO 50 I=1,6,5
      READ 109,P(I,J),P(I+1,J),P(I+2,J),P(I+3,J),P(I+4,J)
   50 PUNCH 109,P(I,J),P(I+1,J),P(I+2,J),P(I+3,J),P(I+4,J)
      C1=P(1,J)
      C2=P(2,J)
      C2=.017453293*C2
      C2=COSF(C2)
      P(11,J)=C1*C2
      IF(J-LM) 55,51,51
   51 C2=1.0-C1*C1
      C3=C1/C2
      C4=SQRTF(C2)
      C4=C3/C4
      C2=C3*C3
      DO 52 I=3,5
      PN(I,J)=P(I,J)/C2
   52 PN(I+3,J)=P(I+3,J)/C4
      PN(9,J)=P(9,J)/C3
      PN(10,J)=P(10,J)/C3
      PN(2,J)=P(2,J)
   55 CONTINUE
      PUNCH 110
      DO 56 I=1,NA,5
      READ 109,PA(I),PA(I+1),PA(I+2),PA(I+3),PA(I+4)
   56 PUNCH 109,PA(I),PA(I+1),PA(I+2),PA(I+3),PA(I+4)
      PUNCH 113
      PUNCH 114
      PUNCH 115
      PUNCH 116
      PUNCH 117
      PUNCH 118
      DO 65 I=2,10
      Y2=P(I,1)
      Y3=P(I,2)
      X2=P(1,1)
      X3=P(1,2)
      DO 65 J=2,NM
      Y1=Y2
      Y2=Y3
      X1=X2
```

```
      X2=X3
      X3=P(1,J+1)
      IF(J-KM) 60,61,62
   60 Y3=P(I,J+1)
      GO TO 64
   61 Y1=PN(I,LM)
      Y2=PN(I,KM)
   62 Y3=PN(I,J+1)
   64 DT1=X2-X1
      DT2=X3-X2
      C1=Y2-Y1
      C2=Y3-Y2
      C3=X3-X1
      C4=DT1/DT2
      A(I,J)=(C2*C4-C1)/(DT1*C3)
      B(I,J)=(C2*C4+C1/C4)/C3
   65 CONTINUE
      Y2=P(1,1)
      Y3=P(1,2)
      X2=P(11,1)
      X3=P(11,2)
      DO 66 J=2,NM
      Y1=Y2
      Y2=Y3
      Y3=P(1,J+1)
      X1=X2
      X2=X3
      X3=P(11,J+1)
      DT1=X2-X1
      DT2=X3-X2
      C1=Y2-Y1
      C2=Y3-Y2
      C3=X3-X1
      C4=DT1/DT2
      A(1,J)=(C4*C2-C1)/(DT1*C3)
   66 B(1,J)=(C4*C2+C1/C4)/C3
      DO 239 K=1,NBE
      READ 105,EB,ATB,CPC
      PUNCH 112
      PUNCH 105,EB,ATB,CPC
      ATR=.017453293*ATB
      DO 238 L=1,NA
      PAN=PA(L)
      PAR=.017453293*PAN
      ECS=COSF(PAR-ATR)
      ECS=1.0-CPC-EB*ECS
      IF(P(11,1)-ECS) 202,205,201
  201 PUNCH 119,L,PAN,ECS
      GO TO 238
  202 IF(1.0-ECS) 203,203,204
  203 PUNCH 120,L,PAN,ECS
      GO TO 238
  204 IF(ECS-P(11,NEP)) 206,200,200
  200 KC=NEP
      GO TO 209
  205 KC=2
      GO TO 209
  206 DO 208 J=2,NEP
      C1=P(11,J)
      IF(ECS-C1) 207,207,208
  207 KC=J
```

```
      GO TO 209
  208 CONTINUE
  209 AR=A(1,KC)
      BR=B(1,KC)
      CR=P(1,KC)
      XR=P(11,KC)
      KL=KC-1
      AL=A(1,KL)
      BL=B(1,KL)
      CL=P(1,KL)
      XL=P(11,KL)
      C1=ECS-XR
      C1=CR+C1*(BR+C1*AR)
      C2=ECS-XL
      C2=CL+C2*(BL+C2*AL)
      IF(KC-2) 210,210,211
  210 C2=C1
      GO TO 213
  211 IF(NEP-KC) 212,212,213
  212 C1=C2
  213 EP=(C1+C2)/2.0
      C2=1.0-EP*EP
      C3=EP/C2
      C4=SQRTF(C2)
      C4=C3/C4
      C2=C3*C3
      DO 234 I=2,10
      AL=A(I,KL)
      BL=B(I,KL)
      XL=P(1,KL)
      AR=A(I,KC)
      BR=B(I,KC)
      XR=P(1,KC)
      IF (I-2) 220,220,221
  220 C1=1.0
      GO TO 226
  221 IF (I-5) 222,222,223
  222 C1=C2
      GO TO 226
  223 IF (I-8) 224,224,225
  224 C1=C4
      GO TO 226
  225 C1=C3
  226 QR=EP-XR
      QR=QR*(BR+QR*AR)
      QL=EP-XL
      QL=QL*(BL+QL*AL)
      IF (KC-LM) 227,227,230
  227 QR=QR+P(I,KC)
  228 QL=QL+P(I,KL)
      IF (KC-2) 229,229,233
  229 QL=QR
      GO TO 233
  230 QR=C1*(QR+PN(I,KC))
      IF (KC-KM) 228,228,231
  231 QL=C1*(QL+PN(I,KL))
      IF (KC-NEP) 233,232,232
  232 QR=QL
  233 Q(I)=(QR+QL)/2.0
  234 CONTINUE
      C2=Q(3)
```

```
      IF (C2) 236,235,236
  235 C1=Q(2)
      GO TO 237
  236 C1=ATANF(Q(6)/C2)
      C1=57.295780*C1
  237 PUNCH 111, PAN,ECS,Q(2)
      PUNCH 106, EP,C1,C2,Q(6)
      PUNCH 106, Q(4),Q(7),Q(9),Q(10)
      PUNCH 106, Q(5),Q(8)
  238 CONTINUE
  239 CONTINUE
      IF (INP) 240,49,240
  240 STOP
  100 FORMAT(49H1PN0131-INTERPOLATION OF PART.ARC DATA J.LUND 8-9,3H-63)
  101 FORMAT(49H0                                                ,3H   )
  102 FORMAT(1XI4,1XI4,1XI4,1XI4,1XI4)
  103 FORMAT(2XI4,6XI4,6XI4,6XI4,6XI4)
  104 FORMAT(9H0NO.PAD E4X5HLIMIT3X8HNO.BRG.E3X7HNO.ANG.5X5HINPUT)
  105 FORMAT(E15.7,E15.7,E15.7)
  106 FORMAT(E15.7,E15.7,E15.7,E15.7)
  107 FORMAT(49H0E ATT FR DFRDE DFRDEW FT DFTDE DFTDEW DFREDA DFT,3HEDA)
  108 FORMAT(3H0E(,I2,1H))
  109 FORMAT(1XE13.6,1XE13.6,1XE13.6,1XE13.6,1XE13.6)
  110 FORMAT(13H0PIVOT ANGLES)
  111 FORMAT(1H0,E14.7,E15.7,E15.7)
  112 FORMAT(//1H0)
  113 FORMAT(14H0OUTPUT FORMAT)
  114 FORMAT(8H EPS.BRG5X7HATT.BRG5X4HCP/C)
  115 FORMAT(38H PIVOT ANG    E*COS(A)     ATT.PAD(INTP))
  116 FORMAT(8H EPS.PAD5X7HATT.PAD5X2HFR8X2HFT)
  117 FORMAT(7H DFR/DE6X6HDFT/DE6X7HDFR/EDA3X7HDFT/EDA)
  118 FORMAT(23H DFR/D(E/W)   DFT/D(E/W))
  119 FORMAT(4H0PAD,I2,17H NO LOAD,PVT.ANG=,E13.6,10H,E*COS(A)=,E13.6)
  120 FORMAT(4H0PAD,I2,13H E=1,PVT.ANG=,E13.6,10H,E*COS(A)=,E13.6)
      END
```

APPENDIX C

COMPUTER PROGRAM PN0091: NUMERICAL CALCULATION OF THE INCOMPRESSIBLE FULL OR PARTIAL JOURNAL BEARING

TECHNICAL REPORT

No. MTI 63TR29

Date Aug. 7, 1963

NUMERICAL CALCULATION OF THE INCOMPRESSIBLE FULL OR PARTIAL JOURNAL BEARING

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ABSTRACT

This report describes the computer program PN0091: "Hydrodynamic Incompressible Journal Bearing." The program exists in 2 versions, one for the IBM 704 computer and one for the IBM 1620 computer. The report contains a description of the analysis and the instructions for using the programs. The program solves the dynamical incompressible Reynolds equation in dimensionless form by finite difference equations and gives results for the pressure distribution, the load and the attitude angle. The rupture of the oil film is included. The bearing may be full 360° or a partial arc.

INTRODUCTION

The dynamical Reynolds equation is a partial differential equation relating the pressure in the fluid film to the journal rotational speed and the squeeze film velocity. The film is assumed to be isothermal so that the viscosity is constant. The equation is used in dimensionless form and is given as Eq. (4) page 4.

The bearing is taken to be operating in air. Therefore, no oil can enter the bearing from the ends and the film pressure cannot be lower than ambient. For this reason the film contracts in the diverging part of the bearing with a uniform pressure of P_a . This is denoted film rupture and is taken into account.

Reynolds equation is transformed into finite difference form and solved by iteration. The convergence of the iteration process is established in two ways, one in which the relative difference between two iterations is checked against a preassigned error and one which compares the result of each iteration against an extrapolated value, i.e., a form of absolute convergence.

The program provides for the calculation of force derivatives with respect to eccentricity ratio and the two squeeze film velocity components. Such derivatives are needed in computing spring and damping coefficients for the bearing.

If desired the program will iterate on the attitude angle in order to determine the attitude angle corresponding to no horizontal force component.

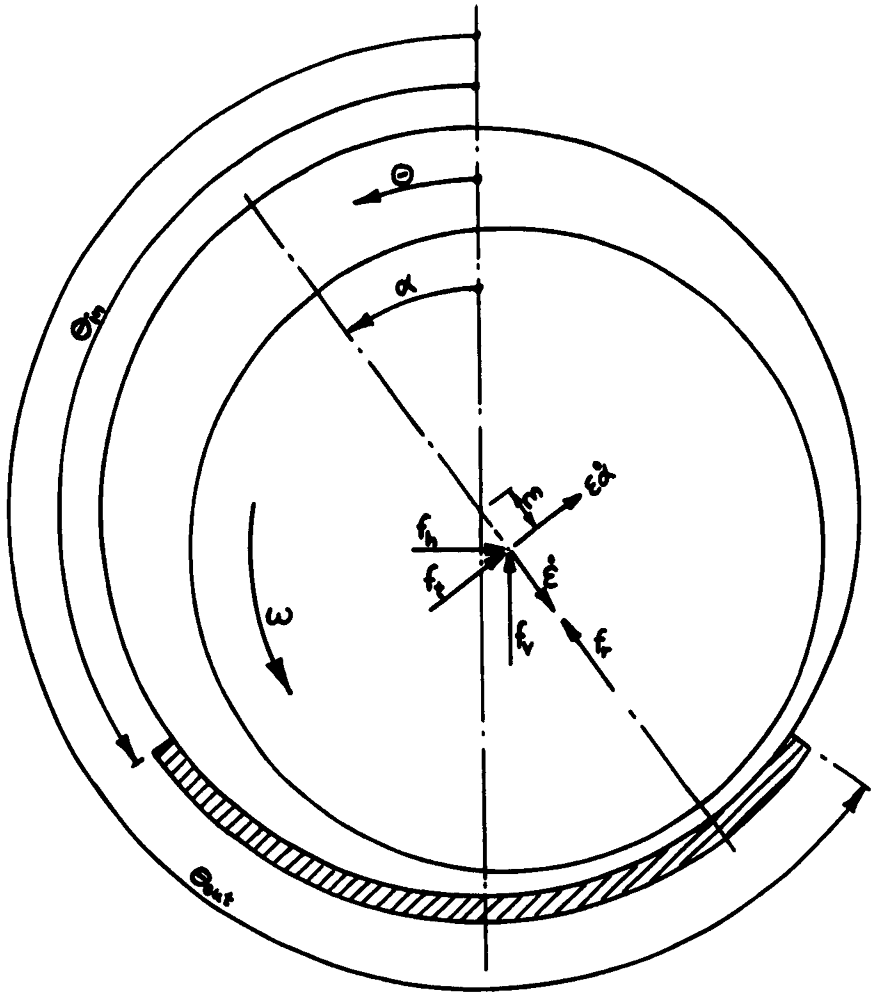

θ
α
θ_{in}
θ_{out}
ω
f_h
f_t
f_v
f_r
$\varepsilon\dot{\alpha}$
$\dot{\varepsilon}$

Fig. 1

ANALYSIS

NOMENCLATURE

C	Diametrical clearance, inch
D	Journal diameter, inch
F	Force, lbs.
f_r, f_t	Radial and tangential force components, dimensionless, see Figure 1.
f_v, f_h	Vertical and horizontal force components, dimensionless, see Figure 1
h	Dimensionless film thickness
i, j	Finite difference coordinates, axial and circumferential, see Figure 2.
k	Iteration number
L	Bearing length, inch
m	Number of circumferential subdivisions, see Figure 2
n	Number of axial subdivisions, see Figure 2
N	Journal speed, RPS
P	Dimensionless pressure (above ambient)
R	Journal radius, inch
S	Sommerfeld number
t	Time, seconds
U_1, U_2	Journal and bearing surface speed, inch/sec.
x, z	Circumferential and axial coordinates, dimensionless
y_k	Sum of all pressures after the k'th iteration, see Eq. (10)
$y_{k\infty}$	Extrapolated pressure sum after the k'th iteration, see Eq. (11)
α	Attitude angle, degrees
$\dot{\alpha}$	Tangential speed of journal center, rad/sec.
δ_A	Absolute convergence limit, see Eq. (12)
δ_R	Relative convergence limit, see Eq. (9)
Δ_A	Error in absolute convergence, see Eq. (12)
Δ_R	Error in relative convergence, see Eq. (9)
ϵ	Eccentricity ratio
$\dot{\epsilon}$	Radial speed of journal center, sec^{-1}
θ	Circumferential angular coordinate, see Figure 1.
μ	Viscosity, lbs-sec/in^2
ω	Angular speed of journal, rad/sec.

For an incompressible lubricant Reynolds equation is:

$$(1)\quad \frac{\partial}{\partial \bar{x}}\left[\frac{\bar{h}^3}{12\mu}\frac{\partial \bar{P}}{\partial \bar{x}}\right] + \frac{\partial}{\partial \bar{z}}\left[\frac{\bar{h}^3}{12\mu}\frac{\partial \bar{P}}{\partial \bar{z}}\right] = \frac{1}{2}\frac{\partial}{\partial \bar{x}}\left[\bar{h}(U_1+U_2)\right] + \frac{\partial \bar{h}}{\partial t}$$

To make dimensionless set:

$$(2)\quad x = \frac{\bar{x}}{D} \qquad z = \frac{\bar{z}}{L} \qquad h = \frac{\bar{h}}{C}, \qquad P = \frac{\bar{P}}{\mu N\left(\frac{R}{C}\right)^2}$$

Furthermore: $U_1 = \pi D N \qquad U_2 = 0$

Also set:

$$(3)\quad h = \frac{1}{2} + \frac{\varepsilon}{2}\cos(\theta-\alpha)$$

where $\theta = \frac{\bar{x}}{R} = 2x$

Under isothermal conditions μ is constant and the dimensionless Reynolds equation becomes:

$$(4)\quad \frac{\partial}{\partial x}\left[h^3\frac{\partial P}{\partial x}\right] + \left(\frac{D}{L}\right)^2\frac{\partial}{\partial z}\left[h^3\frac{\partial P}{\partial z}\right] = 12\pi\left[\frac{\dot{\varepsilon}}{\omega}\cos(\theta-\alpha) + \left(\frac{\varepsilon\dot{\alpha}}{\omega} - \frac{\varepsilon}{2}\right)\sin(\theta-\alpha)\right]$$

The corresponding finite difference equation is:

$$(5)\quad P_{i,j} = \frac{h^3_{i,j+\frac{1}{2}}P_{i,j+1} + h^3_{i,j-\frac{1}{2}}P_{i,j-1} + \left(\frac{D}{L}\right)^2\left(\frac{\Delta x}{\Delta z}\right)^2\left[h^3_{i+\frac{1}{2},j}P_{i+1,j} + h^3_{i-\frac{1}{2},j}P_{i-1,j}\right] - 12\pi\Delta x^2\left[\frac{\dot{\varepsilon}}{\omega}\cos(\theta-\alpha) + \left(\frac{\varepsilon\dot{\alpha}}{\omega} - \frac{\varepsilon}{2}\right)\sin(\theta-\alpha)\right]_{i,j}}{h^3_{i,j+\frac{1}{2}} + h^3_{i,j-\frac{1}{2}} + \left(\frac{D}{L}\right)^2\left(\frac{\Delta x}{\Delta z}\right)^2\left[h^3_{i+\frac{1}{2},j} + h^3_{i-\frac{1}{2},j}\right]}$$

where the finite difference mesh is as follows:

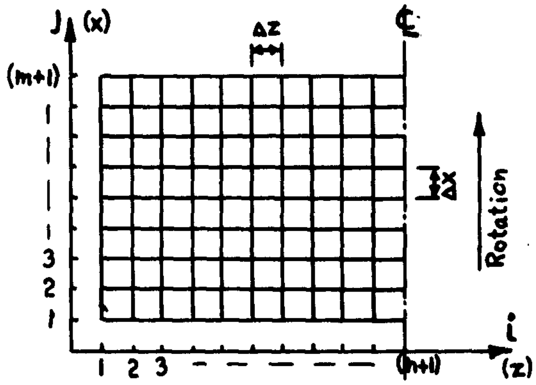


Figure 2.

The pressure at the bearing end is ambient (i.e.0). At the leading and the trailing edge it is given by the input (partial arc only). At the bearing centerline there is symmetry. Thus the boundary conditions to Eq. (5) become:

$$
\begin{aligned}
&P_{1,j} = 0 \\
&P_{n+2,j} = P_{n,j} \\
(6)\qquad &P_{i,1} = \begin{cases} P_{In}\ (\text{input})\ ,\ i=2,3\text{---}(n+1) & \underline{\text{partial arc}} \\ P_{i,m+1} & \underline{\text{full } 360^\circ \text{ bearing}} \end{cases} \\
&P_{i,m+1} = \begin{cases} P_{Out}\ (\text{input})\ ,\ i=2,3,\text{----}(n+1) & \underline{\text{partial arc}} \\ P_{i,1} & \underline{\text{full } 360^\circ \text{ bearing}} \end{cases} \\
&\text{If } (P_{i,j})_{\text{calculated}} < 0\ ,\ \text{then } P_{i,j} = 0
\end{aligned}
$$

The pressure is initially set equal to zero and Eq. (5) is then solved by iterations.

The load carrying capacity is calculated from:

$$
(7)\qquad f_r = -2\int_0^{\frac{l}{2}}\int_{\Theta_{in}}^{\Theta_{out}} P\cos(\Theta-\alpha)\,dx\,dz = -2\Delta x\cdot\Delta z\sum_{i,j} P_{i,j}\cos(\Theta_{i,j}-\alpha)
$$

$$
(8)\qquad f_t = 2\int_0^{\frac{l}{2}}\int_{\Theta_{in}}^{\Theta_{out}} P\sin(\Theta-\alpha)\,dx\,dz = 2\Delta x\cdot\Delta z\sum_{i,j} P_{i,j}\sin(\Theta_{i,j}-\alpha)
$$

The indicated summations are computed by Simpson rule of integration, i.e. $\frac{1}{3}\Delta[P_1+4P_2+2P_3+4P_4+----]$ and if either m or n are odd the first interval is integrated by $\frac{1}{12}\Delta[5P_1+8P_2-P_3]$. In addition end corrections are made at the boundary to the ruptured film.

Iteration Convergence and Extrapolation

The convergence of the pressure iteration is tested by two methods, which shall be denoted relative convergence and absolute convergence. The relative convergence is tested as follows: after the k'th pressure iteration compute:

$$\frac{\sum_i \sum_j \left| P_{i,j}^{(k)} - P_{i,j}^{(k-1)} \right|}{\sum_i \sum_j P_{i,j}^{(k)}} - \delta_R = \Delta_R \tag{9}$$

where δ_R is given by the input. When Δ_R becomes zero or negative then relative convergence has been achieved.

To compute the absolute convergence the following criteria is chosen. After each iteration the sum of the pressures are computed. Let the result for the k'th iteration be denoted:

$$y_k = \sum_i \sum_j P_{i,j}^{(k)} \tag{10}$$

Assume that after infinitely many iterations this sum will be $y_{k\infty}$ and set:

$$y_k = y_{k\infty} - A e^{-Bk} \qquad (A \text{ and } B \text{ constants})$$

from which:

$$y_{k\infty} = y_k + \frac{(y_k - y_{k-1})^2}{2y_{k-1} - y_{k-2} - y_k} \tag{11}$$

$y_{k\infty}$ is calculated after each iteration (exceptions, see later) and the absolute convergence is computed from:

$$\frac{\left| y_{k\infty} - y_k \right|}{y_k} - \delta_A = \Delta_A \tag{12}$$

where δ_A is given by the input. When Δ_A becomes zero or negative absolute convergence has been achieved.

In order to obtain complete convergence, both relative and absolute convergence must be satisfied. Equations (9), (11) and (12) are printed as output after each iteration. Eq. (11) serves an additional purpose, namely to extrapolate the pressure distribution. When y_k becomes a smooth curve and starts to level off then a new extrapolated pressure distribution is calculated from

$$P_{i,j} = \frac{y_{k\infty}}{y_k} \cdot P_{i,j}^{(k)} \tag{13}$$

and the pressure iterations proceed from this new distribution either until convergence is achieved or until y_k again becomes sufficiently smooth that Eq.(13) may be used once more.

It remains to define the criteria for "smoothness". As applied to Eq.(11) $y_{k\infty}$ is not calculated for the first 2 iterations or for the 2 iterations following a pressure extrapolation. In these cases the computer output shows:

$$\text{(14)} \qquad \begin{array}{l} y_{k\infty} = 10^{38} \\ \Delta_A = 1.0 \end{array}$$

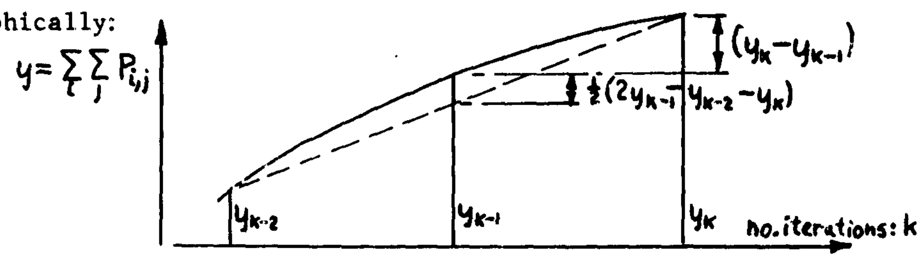
Furthermore, $y_{k\infty}$ is not calculated if the y_k-curve is not monotonely increasing with decreasing gradient, i.e.:

$$\text{(15)} \quad \left.\begin{array}{ll} a) & (2y_{k-1} - y_{k-2} - y_k) > 0 \\ b) & (y_k - y_{k-1}) \geq 10^{-6} \cdot y_k \end{array}\right\} \begin{array}{l} y_{k\infty} \text{ by eq.(11)} \\ \Delta_A \text{ by eq.(12)} \end{array}$$

$$\text{(16)} \quad \left.\begin{array}{ll} a) & k \leq 2 \;, \; (k - k_{extrapolation}) \leq 2 \\ b) & (2y_{k-1} - y_{k-2} - y_k) < 0 \\ c) & (2y_{k-1} - y_{k-2} - y_k) = 0 \,, \; (y_k - y_{k-1}) \neq 0 \\ d) & (2y_{k-1} - y_{k-2} - y_k) > 0 \,, \; (y_k - y_{k-1}) \leq 0 \end{array}\right\} \begin{array}{l} y_{k\infty} = 10^{38} \\ \Delta_A = 1.0 \end{array}$$

$$\text{(17)} \quad \left.\begin{array}{ll} a) & (2y_{k-1} - y_{k-2} - y_k) = 0 \,, \; (y_k - y_{k-1}) = 0 \\ b) & (2y_{k-1} - y_{k-2} - y_k) > 0 \,, \; 0 < (y_k - y_{k-1}) \leq 10^{-6} \cdot y_k \end{array}\right\} \begin{array}{l} y_{k\infty} = y_k \\ \Delta_A = 0.0 \end{array}$$

Graphically:



Similarly, Eq. (13) is not applied until $y_{k\infty}$ is "smooth". Basically this is determined by the input item δ_{EX} which specifies how small the relative difference between two consecutive $y_{k\infty}$ must be for extrapolation to be performed. In addition the $y_{k\infty}$-curve must be increasing. Hence, the criteria for extrapolation becomes:

$$(18)\quad \left.\begin{aligned} &a)\quad 0 \leq \left(\frac{y_{k-1,\infty} - y_{k-2,\infty}}{y_{k-1,\infty}} \right) \leq \delta_{EX} \\ &b)\quad 0 \leq \left(\frac{y_{k\infty} - y_{k-1,\infty}}{y_{k\infty}} \right) \leq \delta_{EX} \\ &c)\qquad \Delta_A \neq 1.0 \\ &d)\quad k \geq 4\ ,\ (k - k_{extrapolation}) \geq 4 \end{aligned} \right\}$$

When these equations are satisfied Eq.(13) is performed, otherwise not.

These Equations (15) to (18) are all used in the 704-version of the program. In the 1620-version Eq. (18,a) is not included and δ_{EX} in Eq. (18,b) is built into the program, given the value $\delta_{EX} = .005$. Furthermore, in Eq. (16) $y_{k\infty} = 10^{38}$ is replaced by $y_{k\infty} = 10^{90}$

Input

The input form is shown on page 17. Below follow the instructions for preparing the input.

Card 1 and 2

These are heading cards to be used for identification and must always be given. Punch text in column 2 to 72 (2 to 52 in 1620 version).

Card 3

L/D Ratio This is the ratio between bearing length and journal diameter.

Inlet Angle Θ_{in}. This is the angle in degrees measured from the vertical reference line to the leading edge of the bearing pad in the direction of journal rotation. For a full 360° bearing Θ_{in} may be any value as long as $\Theta_{out} - \Theta_{in} = 360°$.

Outlet Angle, Θ_{out} This is the angle in degrees measured from the vertical reference line to the trailing edge of the bearing pad in the direction of journal rotation. In a full 360° bearing Θ_{out} may be any value as long as $\Theta_{out} - \Theta_{in} = 360°$.

Inlet Pressure P_{In} This is the dimensionless pressure along the leading edge of the bearing pad. If the actual pressure is P psig, then

$$P_{In} = \frac{P}{\mu N \left(\frac{D}{C}\right)^2}$$

Thus, for ambient pressure $P_{In} = 0$. In the finite difference representation the pressure along the leading edge is set equal to P_{In} except at the end of the bearing (i=1) where the pressure is ambient.

Card 4

Outlet Pressure P_{out} This is the dimensionless pressure along the trailing edge of the bearing pad. For comments, see above.

Relative Convergence Limit δ_R This limit determines when relative convergence has been achieved i.e. when the relative difference between two consecutive pressure iterations is less than δ_R. For details, see Eq. (9) page 6. A typical value is $\delta_R = .001$ or $.002$

<u>Absolute Convergence Limit δ_A</u> This limit determines when absolute convergence has been achieved, i.e. when the difference between a given pressure iteration and the corresponding extrapolated final value becomes less than δ_A . For details, see Eq. (12), page 6. A typical value may be $\delta_A = .002$ to $.005$. It should be noted that absolute convergence is much more strict than relative convergence i.e. δ_A should be 2 or 3 times greater than δ_R for consistency. Hence, $\delta_A = .002$ is rather small, especially if the mesh has many points (150 to 200 and more). Considering that $\delta_A = .002$ means an absolute accuracy of .2 percent which in most cases is much finer than needed, and also that the method of solution is approximate anyway, δ_A should not be made any smaller than actually needed. The number of pressure iterations increases rapidly with decreasing δ_A and for most practical cases $\delta_A = .005$ should give sufficient accuracy.

<u>Relaxation Factor for Pressure Calculation f_P</u> In order to increase (or decrease) the rate of convergence of the pressure iterations a relaxation factor may be used. To illustrate assume that in the k'th iteration the pressure $P_{i,j}^{(k)}$ has been computed from Eq. (5) for any meshpoint. Then the actually stored value is not $P_{i,j}^{(k)}$ but

$$P_{i,j} = P_{i,j}^{(k-1)} + f_p \left(P_{i,j}^{(k)} - P_{i,j}^{(k-1)} \right)$$

Hence, $f_P > 1$ will accelerate the convergence and $f_P < 1$ will decelerate the convergence. Setting $f_P = 1$ results in no relaxation. It should be noted that the program includes an additional feature for speeding up the calculation, namely the pressure extrapolation given by Eq. (13). If extrapolation is desired it is questionable if relaxation should also be used.

<u>Card 5</u>

1. <u>Number of Circumferential Subdivisions, m</u> As shown in Figure 2, the circumferential length of the bearing is subdivided into m increments $\Delta X = \frac{\pi}{180} \frac{(\Theta_{out} - \Theta_{in})}{m}$. Thus the number of meshpoints in the circumferential direction is (m+1).
2. <u>Number of Axial Subdivisions, n</u> As shown on Figure 2, the bearing half-length is subdivided into n increments $\Delta Z = \frac{1}{2n}$. Thus the number of meshpoints in the axial direction is (n+1).
3. <u>Full or Partial Bearing</u> If this item is 0 the program assumes that the bearing is full 360° such that the pressure along the leading edge is equal

to the pressure along the trailing edge with a continuous gradient. Hence, item 4, Card 3, and item 1, Card 4, are ignored.

If this item is 1 the program assumes that the bearing is partial arc.

4. Number of Pressure Iterations In order to ensure that the computer does not use an excessive amount of time to meet the convergence limits this item sets a limit for the maximum number of pressure iterations. If this number is exceeded the computer gives the results obtained up to this point.

5. Number of ε-α-Input Frequently the load and other quantities are wanted as a function of eccentricity ratio ε keeping other parameters such as L/D ratio and arc length the same. The program provides for giving as many values of ε as desired and the present item specifies how many cases there are, i.e. how many of Card 6 (or of pairs of Card 6-Card 7).

6. Input Control If this item is zero the program will return after completed calculation to read in an additional input set. If this item is 1 the program assumes this to be the last input set and will go to normal stop after completed calculation. Note, that this of course does not refer to the above mentioned ε-α-input which is regarded as a part of an input set. An input set starts with Card 1.

7. Extrapolation Limit, δ_{EX} (704-version only). After each pressure iteration, say the k'th, all pressures are summed up. Denote the sum y_k. Based on 3 consecutive values of y_k an exponential extrapolation is used to calculate the value of the sum after infinitely many iterations, denoted $y_{k\infty}$. When the relative difference between three consecutive values of $y_{k\infty}$ is less than δ_{EX} an extrapolation of the pressure distribution is performed by multiplying each pressure by $y_{k\infty}/y_k$. For details, see Eq. (13) and (18), page 6 and 8. A few remarks are needed. It is obvious that the intention of the extrapolation is to speed up the convergence of the pressure iteration but it is difficult to ascertain that this is always the case. When the absolute convergence limit δ_A is not too tight the extrapolation may reduce the number of iterations by more than 50 percent but when δ_A is small it is more difficult to evaluate the effect. After an extrapolation the pressure distribution is no longer in "equilibrium" and this reflects clearly in the pressure sums y_k and

very strongly in the extrapolated sums $y_{k\infty}$. In general y_k will be decreasing right after an extrapolation such that $y_{k\infty}$ cannot be computed and therefore is given the arbitrary value 10^{38} (10^{90} in 1620-verson) in the output . After a certain number of iterations y_k will slowly start to increase again and, when smooth enough, a new extrapolation will be performed, repeating the cycle. When δ_A is small and the number of meshpoints is large these cycles may be rather long which tends to reduce the benefit of the extrapolation. If it is not desired to extrapolate, set δ_{EX}=0.0. If extrapolation is desired, an example of its value is δ_{EX} = .005.

Card 6

There should be as many Card 6 as given by item 5, Card 5. Each Card 6 with MATT=±1 (item 5 below) should be followed by a Card 7.

Eccentricity Ratio, ε This is the ratio between the journal eccentricity and the radial clearance, see Figure 1.

Attitude Angle, α This is the angle in degrees measured from the vertical reference line to the line of centers (i.e. the line connecting bearing and journal centers) in the direction of rotation, see Figure 1.

Radial Squeeze Film Velocity, $\dot{\varepsilon}/\omega$ This is the dimensionless velocity of the journal center along the line of centers, see Figure 1. However, if item 5, Card 6, is MATT=-1 then this item means $\Delta(\frac{\dot{\varepsilon}}{\omega})$ for use in calculating force derivatives with respect to $\frac{\dot{\varepsilon}}{\omega}$.

Tangential Squeeze Film Velocity, $\varepsilon\dot{\alpha}/\omega$ This is the dimensionless velocity of the journal center perpendicular to the line of centers, see Figure 1.

MATT If MATT=-1 a Card 7 must be given. The program will perform a number of complete pressure calculations, changing the attitude angle in order to make the horizontal force component zero . Thereafter an additional 4 complete calculations will be performed to calculate $\partial f/\partial \varepsilon$ and $\partial f/\partial(\frac{\dot{\varepsilon}}{\omega})$

If Matt=0 no Card 7 is required. Only one complete calculation will be performed based on the input value for the attitude angle.

If MATT=+1 a Card 7 is required after Card 6. The program will iterate on the attitude angle in order to eliminate the horizontal force component. However, force derivatives will not be computed.

Card 7

Card 7 can only be given when MATT=± 1 (see item 5 above)

Eccentricity Ratio Increment $\Delta\varepsilon$ When MATT=+1 this item is not used. When MATT=-1 $\Delta\varepsilon$ is used in computing the derivatives with respect to ε. Upon completion of the attitude angle iterations the program proceeds to calculate the change in bearing load as a function of ε, keeping all other parameters constant. Denoting the original eccentricity ratio ε_0 (item 1, Card 6) the program performs calculations for $\varepsilon=\varepsilon_0+\Delta\varepsilon$ and for $\varepsilon=\varepsilon_0-\Delta\varepsilon$. Hence, the results may be used to calculate $\partial f/\partial\varepsilon = \Delta f/2\Delta\varepsilon$

Convergence Limit for Attitude Angle Iteration, δ_α. δ_α is in degrees and determines when the attitude angle iteration has converged. After the k'th iteration the program computes the value of the attitude angle as

$$\alpha^{(k+1)} = \tan^{-1}(f_t/f_r)$$

If $|\alpha^{(k+1)} - \alpha^{(k)}| \leq \delta_\alpha$ the calculation has converged (or if item 4 below is exceeded) and the program proceeds to calculate derivatives. Otherwise a new trial value of α is determined as described below and a new calculation is performed. A typical value is $\delta_\alpha = .01^\circ$

Relaxation Factor for Attitude Angle Iteration f_α As described above, a new pressure calculation is performed when $|\alpha^{(k+1)}-\alpha^{(k)}|>\delta_\alpha$. This new calculation is based on an α-value determined as follows: the first calculation uses the original value for α (item 2, Card 6). The two next calculations use

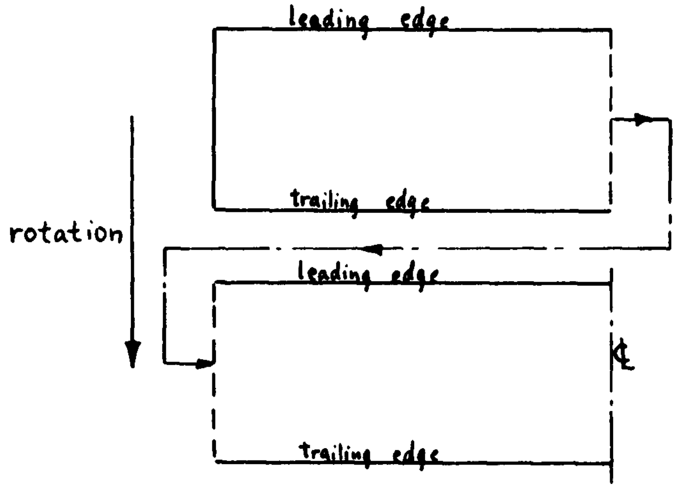
$$\alpha_{k+1} = \alpha^{(k)} + f_\alpha\cdot(\alpha^{(k+1)} - \alpha^{(k)})$$

Any additional calculations uses an α-value determined by parabolic interpolation. f_α is actually a function of eccentricity ratio, L/D-ratio, bearing geometry etc. It usually has a maximum around ε =.5 where f_α = 2.5 to 3. In general, use f_α =1.0, to 1.5.

Maximum Number of Attitude Angle Iterations This item limits the number of attitude angle iterations so that an excessive number of iterations is not performed even if convergence has not been obtained. Normally 5 to 6 iterations are needed if δ_α =.01°.

<u>Output</u>

The output is in general self-explanatory. First come the two heading cards followed by 3 or 4 lines listing the input. Thereafter the convergence of the pressure iteration is printed as a list with 6 columns. The first column gives the number of the iteration, the second column is Δ_R from Eq. (9), the third column is Δ_A from Eq. (12), the fourth column is y_k from Eq.(10), the fifth column is the extrapolated sum $y_{k\infty}$ from Eq. (11) or from Eq. (15) to (17) and the sixth column gives a count of the number of iterations after last pressure extrapolation. In the 1620-version column 5 and 6 are interchanged. Note, that for convergence Δ_R and Δ_A must be zero or negative. Next follows the pressure distribution listed as:



The output format provides for a maximum of 7 columns. Hence, if (n+1) exceeds 7 the pressure field is broken up as shown above. The last line gives the dimensionless radial force component f_r (Eq.(7)), the tangential force component f_t (Eq.(8)), the vertical force component f_v, the horizontal force component f_h, the total force f_{total}, the Sommerfeld Number S and the calculated attitude angle, α_{calc} where:

$$f_v = -2\int_0^{\frac{1}{2}}\int_{\theta_{in}}^{\theta_{out}} P\cos\theta \, dx\, dz = f_r\cos\alpha + f_t\sin\alpha$$

$$f_h = 2\int_0^{\frac{1}{2}}\int_{\theta_{in}}^{\theta_{out}} P\sin\theta \, dx\, dz = -f_r\sin\alpha + f_t\cos\alpha$$

$$f_{total} = \sqrt{f_r^2 + f_t^2}$$

$$S = \frac{1}{f_{total}} = \frac{\mu N}{F/DL}\left(\frac{D}{C}\right)^2$$

$$\alpha_{calc} = \tan^{-1}\left(f_t/f_r\right)$$

Hence, to convert the dimensionless force f to actual force F;

$$F = \mu D L N \left(\frac{D}{C}\right)^2 \cdot f \quad \text{lbs}$$

Similarly, the actual pressure $\bar{P}$ in psig is obtained from the dimensionless pressure P by

$$\bar{P} = \mu N \left(\frac{D}{C}\right)^2 \cdot P \quad \text{psig}$$

Computer Operation

The input is on cards. In the 704-version the output is given on Tape 3, which is the only tape used by the program. Upon completion of calculation Tape 3 is given an "End of File", but not rewound, and the computer stops with 77777_8 in the address field. The stop is at 3361_8. The program is compiled with part of LSTG's TOP-system, i.e. senseswitch 1 and 2 down. The 1620-version is written in FORTRAN I, both input and output is on cards.

Computer Time

The two most important factors in determining the computer time is the number of meshpoints m x n and the absolute convergence limit δ_A . Whereas, it is difficult to evaluate the effect of the latter, it is approximately correct to state that the computer time is proportional to $(m \times n)^2$. This is due to two contributions: a) for each iteration the time is of course directly proportional to the number of meshpoints. This time may be estimated from:

1000 mesh point calculations (i.e.of $P_{i,j}$) per 5 seconds,

b) the number of iterations needed for convergence increases approximately proportional to the number of meshpoints. Example: for δ_A = .002 and δ_{EX} = .005 the average number of iterations is:

	No. iterations	time
m x n = 12 x 6	46.6	17 seconds
m x n = 16 x 6	65	31 seconds
m x n = 36 x 10	Approx.200	6 minutes

The first two numbers are averaged over several values of ϵ . It should be noted that ϵ also affects the number of iterations. As an example, take the same conditions as above. Then:

	m x n = 12 x 6 No. iterations	m x n = 16 x 6 No iterations
$\epsilon = .1$	44	70
$\epsilon = .2$	48	75
$\epsilon = .4$	54	74
$\epsilon = .6$	47	55
$\epsilon = .8$	31	55

From the above both computer time and number of iterations may be estimated.

INPUT FOR PN0091 (IBM 704)

HYDRODYNAMIC INCOMPRESSIBLE JOURNAL BEARING

Card 1. Text, Col. 2-72: ____________________

____

Card 2. Text, Col. 2-72: ____________________

____

Card 3
(1P4E15.7)

_ . _ _ _ _ _ _ _ _ E _ _ _ L/D -Ratio

_ . _ _ _ _ _ _ _ _ E _ _ _ θ_{in}, Inlet Angle, degrees

_ . _ _ _ _ _ _ _ _ E _ _ _ θ_{out}, Outlet Angle, degrees

_ . _ _ _ _ _ _ _ _ E _ _ _ P_{in}, Inlet Pressure, dimensionless

Card 4
(1P4E15.7)

_ . _ _ _ _ _ _ _ _ E _ _ _ P_{out}, Outlet Pressure, dimensionless

_ . _ _ _ _ _ _ _ _ E _ _ _ δ_R, Relative Convergence Limit

_ . _ _ _ _ _ _ _ _ E _ _ _ δ_A, Absolute Convergence Limit

_ . _ _ _ _ _ _ _ _ E _ _ _ f_p, Relaxation Factor for Pressure Calculation.

Card 5
(6I5,1PE15.7)

_ _ _ 1. m, Number of Circumferential Subdivision ($m \leq 200$)

_ _ _ 2. n, Number of Axial Subdivision on Half length ($n \leq 100$)

_ 3. 0: Full 360° Bearing 1: Partial Arc Bearing

_ _ _ 4. Maximum Number of Pressure Iterations

_ _ 5. Number of ε-α-Input

_ 6. 0: More Input sets follow 1: Last Input set

_ . _ _ _ _ _ _ _ _ E _ _ _ 7. δ_{EX}, Extrapolation Limit

Card 6
(1P4E15.7, I5)

_ . _ _ _ _ _ _ _ _ _ E _ _ _ ε, Eccentricity Ratio

_ . _ _ _ _ _ _ _ _ _ E _ _ _ α, Attitude Angle, degrees

_ . _ _ _ _ _ _ _ _ _ E _ _ _ $\frac{\dot{\varepsilon}}{\omega}$, Radial Squeeze Film Velocity,dimensionless

_ . _ _ _ _ _ _ _ _ _ E _ _ _ $\frac{\varepsilon\dot{\alpha}}{\omega}$, Tangential Squeeze Film Velocity,dimensionless

_ MATT, -1: Attitude Angle Iteration + Force Derivatives

0: Single Calculation

+1: Attitude Angle Iteration, no Derivatives

Card 7
(1P3E15.7, I5)
Only if Card 6, item 5, MATT=$\pm$1

_ . _ _ _ _ _ _ _ _ _ E _ _ _ $\Delta\varepsilon$, Eccentricity Ratio Increment for $\frac{\partial f}{\partial \varepsilon}$

_ . _ _ _ _ _ _ _ _ _ E _ _ _ δ_α, Convergence Limit for Attitude Angle Iteration, degrees.

_ . _ _ _ _ _ _ _ _ _ E _ _ _ f_α, Relaxation Factor for Attitude Angle Iteration

_ _ Maximum Number of Attitude Angle Iterations.

Note: An Input Set consists of Card 1 to Card 5 plus as many pairs of Card 6-Card 7 as given by item 5, Card 5.

INPUT FOR PN0091 (IBM 1620)

HYDRODYNAMIC INCOMPRESSIBLE JOURNAL BEARING

Card 1. Text, Col. 2-52: ______________________________

____

Card 2. Text, Col. 2-52: ______________________________

____

Card 3

(1XE14.7, 1XE14.7, 1XE14.7, 1XE14.7)

_ . _ _ _ _ _ _ _ _ _ E _ _ _ L/D -Ratio

_ . _ _ _ _ _ _ _ _ _ E _ _ _ θ_{in}, Inlet Angle, degrees

_ . _ _ _ _ _ _ _ _ _ E _ _ _ θ_{out}, Outlet Angle, degrees

_ . _ _ _ _ _ _ _ _ _ E _ _ _ P_{In}, Inlet Pressure, dimensionless

Card 4

(1XE14.7, 1XE14.7, 1XE14.7, 1XE14.7)

_ . _ _ _ _ _ _ _ _ _ E _ _ _ P_{out}, Outlet Pressure, dimensionless

_ . _ _ _ _ _ _ _ _ _ E _ _ _ δ_R, Relative Convergence Limit

_ . _ _ _ _ _ _ _ _ _ E _ _ _ δ_A, Absolute Convergence Limit

_ . _ _ _ _ _ _ _ _ _ E _ _ _ f_p, Relaxation Factor for Pressure Calculation.

Card 5

(1XI4, 1XI4, 1XI4, 1XI4, 1XI4, 1XI4)

_ _ 1. m, Number of Circumferential Subdivision ($m \leq 26$)

_ _ 2. n, Number of Axial Subdivision on Half length ($n \leq 12$)

_ 3. 0: Full 360° Bearing 1: Partial Arc Bearing

_ _ 4. Maximum Number of Pressure Iterations

_ 5. Number of ε-α-Input

_ 6. 0: More Input Sets follow 1: Last Input Set

Card 6
(1XE14.7, 1XE14.7, 1XE14.7, 1XE14.7, 1XI4)

_ . _ _ _ _ _ _ _ _ _ E _ _ _ ε , Eccentricity Ratio
_ . _ _ _ _ _ _ _ _ _ E _ _ _ α , Attitude Angle, degrees
_ . _ _ _ _ _ _ _ _ _ E _ _ _ $\frac{\dot{\varepsilon}}{\omega}$, Radial Squeeze Film Velocity, dimensionless
_ . _ _ _ _ _ _ _ _ _ E _ _ _ $\frac{\varepsilon\dot{\alpha}}{\omega}$, Tangential Squeeze Film Velocity, "
_ MATT, -1: Attitude Angel Iteration + Force Derivatives.
0: Single Calculation.
+1: Attitude Angle Iteration, no Derivatives.

Card 7
(1XE14.7, 1XE14.7, 1XE14.7, 1XI4)
Only if Card 6, item 5, Matt=$\pm$1

_ . _ _ _ _ _ _ _ _ _ E _ _ _ $\Delta\varepsilon$, Eccentricity Ratio Increment for $\frac{\partial f}{\partial \varepsilon}$
_ . _ _ _ _ _ _ _ _ _ E _ _ _ δ_α , Convergence Limit for Attitude Angle Iteration, degrees
_ . _ _ _ _ _ _ _ _ _ E _ _ _ ξ_α , Relaxation Factor for Attitude Angel Iteration
_ _ Maximum Number of Attitude Angle Iterations

Note: An Input Set consists of Card 1 to Card 5 plus as many pairs of Card 6 - Card 7 as given by item 5, Card 5.

TABLE I

$\frac{L}{D}$	$\frac{L}{B}$	S	ϵ	α	f_r	$\frac{\partial f_r}{\partial\epsilon}$	$\frac{\partial f_r}{\partial(\frac{1}{\omega})}$	f_t	$\frac{\partial f_t}{\partial\epsilon}$	$\frac{\partial f_t}{\partial(\frac{1}{\omega})}$	$\frac{\partial f_r}{\epsilon\partial\alpha}$	$\frac{\partial f_t}{\epsilon\partial\alpha}$
25° ARC		1640.0	.01	84.79	5.55×10^{-5}	.00558	.00161	6.09×10^{-4}	.0610	.0111	-.0601	.00937
		159.0	.1	65.00	.00266	.0302	.0257	.00571	.0645	.0533	-.0514	.0370
		70.7	.2	50.81	.00894	.0641	.0744	.011	.0781	.0893	-.0585	.0414
.25	1.15	29.5	.35	37.04	.027	.167	.208	.0204	.125	.154	-.0198	.103
		12.5	.5	28.17	.0706	.470	.542	.0378	.249	.282	.0226	.183
		4.53	.65	21.22	.206	1.75	1.70	.0799	.667	.633	.187	.401
		1.06	.8	15.03	.906	12.3	9.22	.243	3.22	2.28	1.32	1.36
		.0459	.95	8.34	21.5	1000.0	428.0	3.16	134.0	44.9	64.7	19.9
50° ARC		476.0	.01	87.38	9.55×10^{-5}	.00979	.0131	.0021	.21	.0177	-.203	.013
		45.8	.1	71.23	.00703	.0799	.0633	.0207	.228	.14	-.194	.0813
		20.4	.2	56.84	.0268	.186	.2	.0410	.275	.268	-.171	.155
.25	.573	8.63	.35	42.78	.0851	.502	.593	.0787	.445	.487	-.146	.272
		3.76	.5	33.29	.222	1.37	1.56	.146	.85	.886	-.0874	.472
		1.44	.65	25.88	.625	4.87	4.92	.303	2.2	1.94	.19	.934
		.383	.8	19.68	2.46	29.2	24.8	.879	9.46	6.0	2.31	2.37
		.0262	.95	12.15	37.3	1410.0	611.0	8.04	247.0	76.0	27.3	12.3
		209.0	.01	86.76	2.69×10^{-4}	.0274	.0219	.00478	.479	.0532	-.467	.0402
		20.3	.1	67.19	.0191	.217	.185	.0454	.505	.381	-.413	.256
		9.16	.2	52.14	.067	.476	.565	.0862	.597	.669	-.317	.443
		3.91	.35	38.44	.2	1.2	1.57	.159	.919	1.14	-.139	.741
		3.0	.4	35.18	.272	1.61	2.12	.192	1.09	1.37	-.0448	.89
.5	1.15	1.73	.5	29.68	.501	3.17	3.93	.286	1.73	2.0	.165	1.22
		.951	.6	25.13	.952	6.76	7.81	.447	3.0	3.17	.707	1.88
		.678	.65	23.11	1.36	10.6	11.6	.579	4.26	4.18	1.26	2.38
		.188	.8	17.63	5.06	59.1	53.9	1.61	17.2	12.5	7.48	6.02
		.0158	.95	11.79	62.0	2111.0	1040.0	12.9	365.0	129.0	68.8	27.7
		157.0	.01	86.16	4.25×10^{-4}	.0431	.0271	.00636	.637	.0845	-.623	.0667
		15.3	.1	65.0	.0276	.315	.287	.0591	.663	.551	-.518	.388
		6.97	.2	49.92	.0924	.662	.831	.11	.77	.923	-.359	.643
.75	1.72	3.0	.35	36.65	.267	1.61	2.21	.199	1.16	1.53	-.079	1.03
		1.34	.5	28.25	.656	4.17	5.4	.353	2.15	2.62	.459	1.7
		.53	.65	22.03	1.75	13.6	15.6	.708	5.2	5.38	2.17	3.24
		.15	.8	16.90	6.36	72.5	70.1	1.93	20.3	15.9	11.0	8.08
		.0136	.95	11.7	71.9	2360.0	1230.0	14.9	407.0	150.0	91.9	34.6
60° ARC		371.0	.01	87.85	1.01×10^{-4}	.0105	.0248	.00269	.269	.0167	-.256	.0124
		35.8	.1	72.11	.00859	.0981	.0876	.0266	.289	.173	-.246	.0923
		16.0	.2	58.45	.0327	.228	.25	.0533	.352	.324	-.224	.178
.25	.477	6.79	.35	44.54	.105	.612	.740	.103	.564	.603	-.191	.313
		3.01	.5	35.0	.272	1.66	1.94	.19	1.07	1.08	-.11	.533
		1.19	.65	27.76	.746	5.71	5.98	.393	2.7	2.28	.224	.96
		.333	.8	21.67	2.79	31.6	28.8	1.11	11.5	6.94	2.18	2.17
		.0249	.95	13.11	39.1	1440.0	620.0	9.11	266.0	81.7	20.0	9.39

TABLE I (Cont.)

$\frac{L}{D}$	$\frac{L}{B}$	S	ϵ	α	f_r	$\frac{\partial f_r}{\partial \epsilon}$	$\frac{\partial f_r}{\partial(\frac{\dot\epsilon}{\omega})}$	f_t	$\frac{\partial f_t}{\partial \epsilon}$	$\frac{\partial f_t}{\partial(\frac{\dot\epsilon}{\omega})}$	$\frac{\partial f_r}{\epsilon \partial \alpha}$	$\frac{\partial f_t}{\epsilon \partial \alpha}$
60° ARC (Cont'd)		147.0	.01	87.33	3.18×10^{-4}	.0327	.0452	.00682	.683	.0597	-.659	.044
		14.2	.1	68.43	.0259	.295	.261	.0655	.73	.52	-.591	.336
		6.42	.2	53.9	.0918	.647	.764	.126	.854	.917	-.476	.584
.5	.955	2.77	.35	40.26	.276	1.6	2.12	.233	1.29	1.57	-.266	.963
		1.25	.5	31.38	.684	4.21	5.3	.417	2.41	2.73	.144	1.58
		.502	.65	24.84	1.81	13.4	15.4	.836	5.7	5.54	1.43	2.85
		.149	.8	19.54	6.34	68.8	68.5	2.25	21.8	15.9	8.6	6.32
		.0143	.95	12.78	68.0	2150.0	1110.0	15.4	394.0	141.0	47.0	23.1
		103.0	.01	87.17	4.78×10^{-4}	.049	.0543	.00968	.969	.0935	-.941	.0686
		10.1	.1	66.49	.0396	.452	.411	.091	1.01	.792	-.804	.545
		4.6	.2	51.32	.136	.967	1.05	.17	1.17	1.36	-.573	.922
.75	1.43	2.0	.35	38.09	.393	2.32	3.2	.308	1.74	2.24	-.182	1.46
		.913	.5	29.69	.951	5.85	7.75	.542	3.15	3.8	.547	2.34
		.373	.65	23.55	2.45	18.2	21.9	1.07	7.33	7.6	2.72	4.18
		.115	.8	18.34	8.25	87.3	93.2	2.77	26.6	20.4	13.7	9.1
		.0121	.95	12.68	80.3	2420.0	1320.0	18.1	440.0	166.0	63.5	30.1
<u>80° ARC</u>		90.7	.01	87.91	4.02×10^{-4}	.0429	.137	.011	1.1	.0621	-1.03	.05
		8.79	.1	71.31	.0365	.424	.431	.108	1.17	.729	-.951	.435
		3.99	.2	57.35	.135	.95	1.16	.211	1.37	1.35	-.796	.788
.5	.716	1.76	.35	43.94	.41	2.34	3.17	.395	2.05	2.34	-.512	1.28
		.821	.5	35.14	.995	5.76	7.85	.701	3.61	3.95	.0624	1.94
		.351	.65	28.81	2.49	17.4	22.0	1.37	8.31	7.52	1.82	2.99
		.116	.8	23.33	7.94	78.8	80.2	3.42	28.4	20.6	5.86	5.47
		.013	.95	14.27	74.5	2220.0	1140.0	19.0	433.0	154.0	24.4	15.0
		57.4	.01	87.73	6.92×10^{-4}	.073	.179	.0174	1.74	.114	-1.65	.0931
		5.61	.1	68.76	.0645	.745	.738	.166	1.81	1.29	-1.45	.849
		2.57	.2	54.47	.226	1.59	2.01	.316	2.08	2.26	-1.1	1.44
.75	1.07	1.15	.35	41.37	.654	3.71	5.28	.576	3.01	3.73	-.488	2.24
		.901	.4	38.17	.873	4.85	7.05	.686	3.49	4.39	-.211	2.56
		.546	.5	33.0	1.54	8.87	12.5	.998	5.21	6.13	.622	3.34
		.321	.6	28.89	2.73	17.2	23.4	1.5	8.4	8.96	2.2	4.46
		.24	.65	27.27	3.71	24.9	33.4	1.92	11.1	11.3	3.5	5.19
		.0832	.8	22.50	11.1	103.0	116.0	4.6	36.4	27.7	11.2	8.87
		.0108	.95	14.29	89.8	2480.0	1370.0	22.9	492.0	186.0	31.6	21.2
		45.5	.01	87.46	9.73×10^{-4}	.102	.203	.022	2.2	.171	-2.09	.155
		4.46	.1	67.49	.0859	.989	.979	.207	2.27	1.72	-1.78	1.18
		2.06	.2	52.84	.293	2.06	2.69	.387	2.56	2.93	-1.25	1.96
1.0	1.43	.931	.35	39.89	.824	4.67	6.86	.689	3.61	4.71	-.329	2.96
		.488	.5	31.90	1.9	10.8	15.8	1.18	6.13	7.58	1.15	4.33
		.20	.65	26.33	4.48	29.5	41.1	2.22	13.2	13.7	4.98	6.66
		.0712	.8	21.98	13.0	118.0	139.0	5.25	41.3	32.4	15.6	11.1
		.00992	.95	14.29	97.7	2640.0	1490.0	24.9	514.0	202.0	36.2	24.7

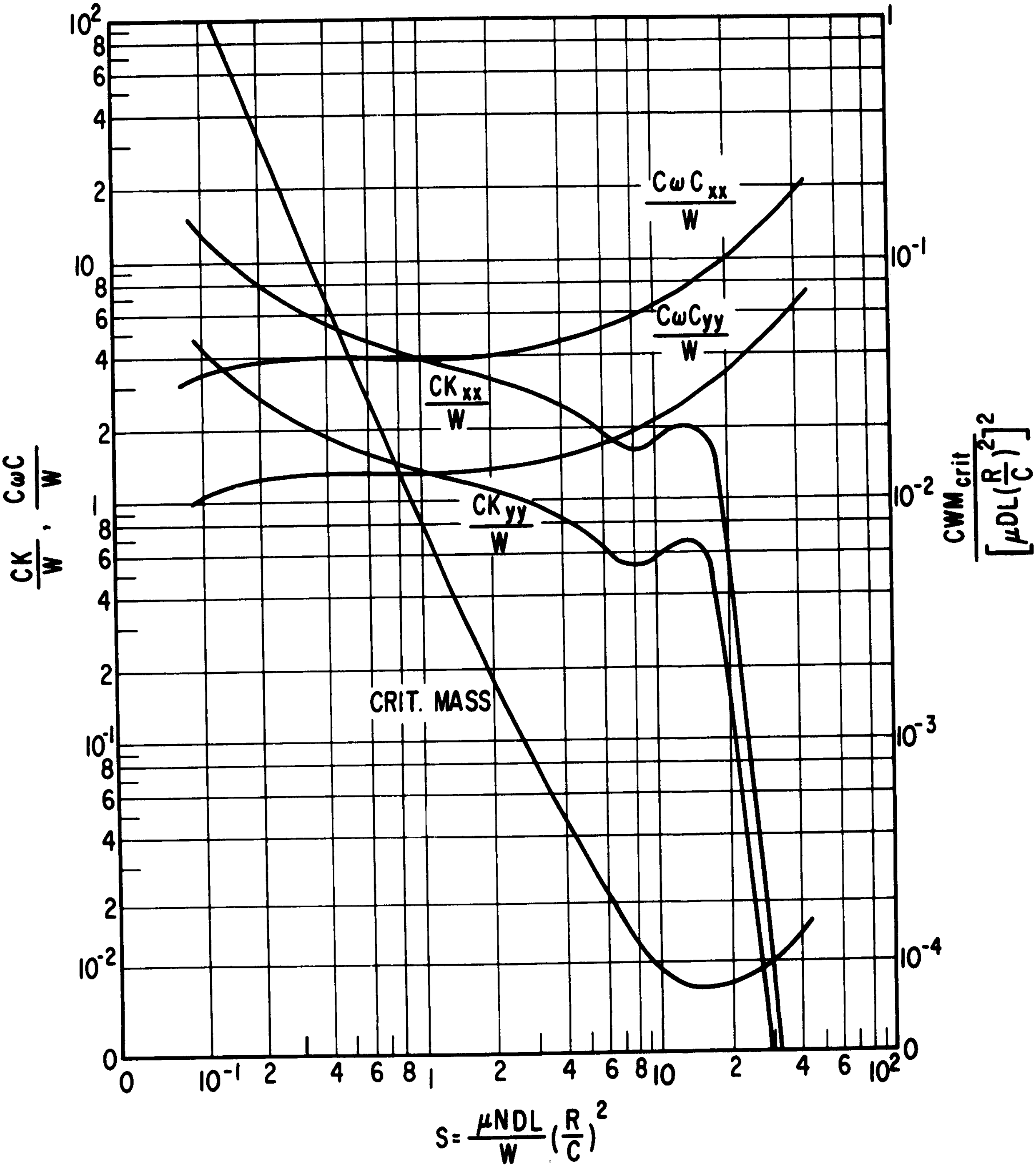


Fig. 1 Six 50° Tilting Pads, Centrally pivoted, L/D = 25, L/B = .573, C'/C = 1. Load between pads. No pad intertia.

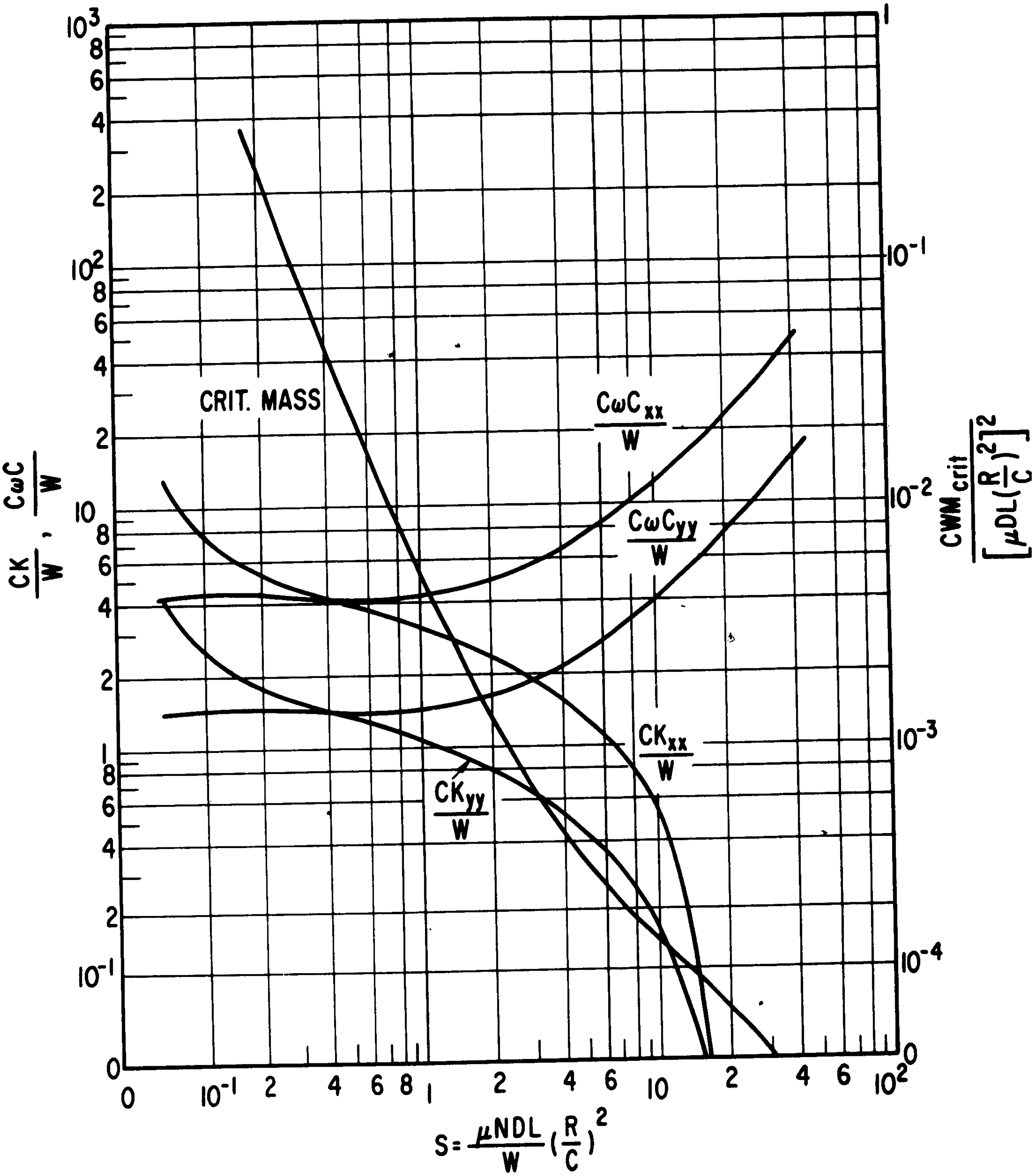


Fig. 2 Six 50° Tilting Pads, Centrally pivoted, L/D = .5, L/B = 1.146, C'/C = 1. Load between pads. No pad inertia.

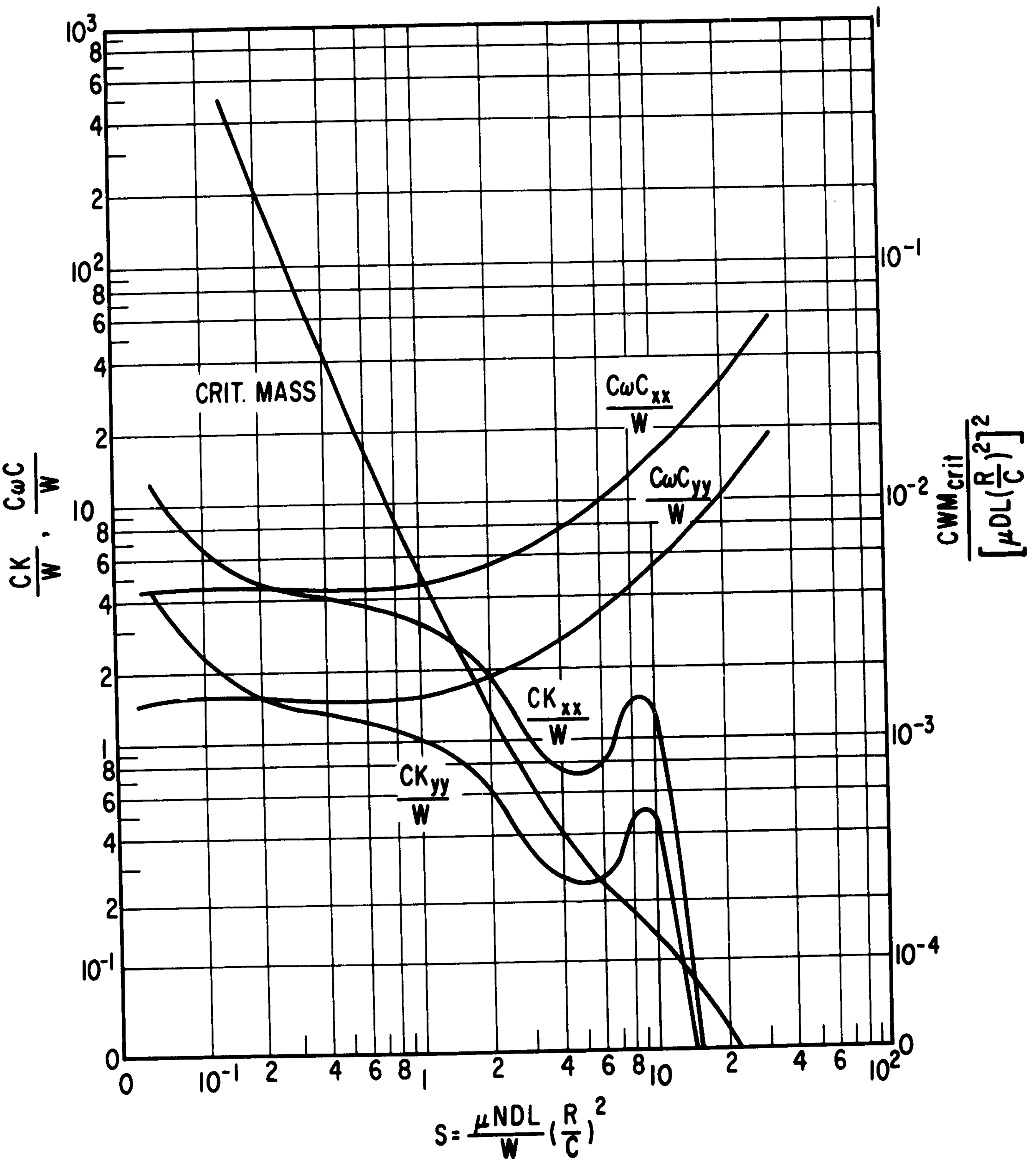


Fig. 3 Six 50° Tilting Pads, Centrally pivoted, L/D = .75, L/B = 1.719, C'/C = 1. Load between pads. No pad inertia.

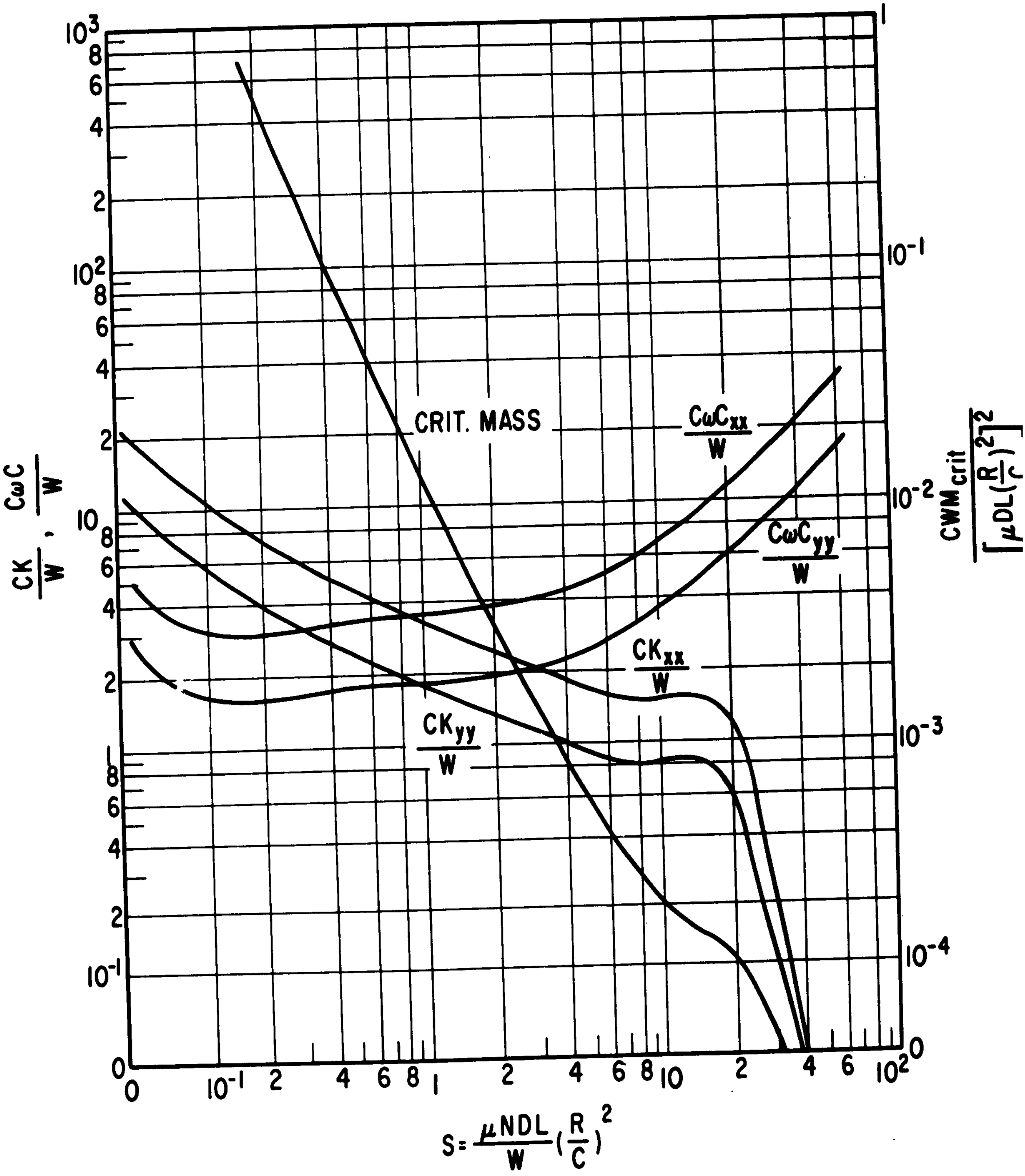


Fig. 4 Five 60° Tilting Pads, Centrally pivoted, L/D = .25, L/B = .477, C'/C = 1. Load between pads. No pad inertia.

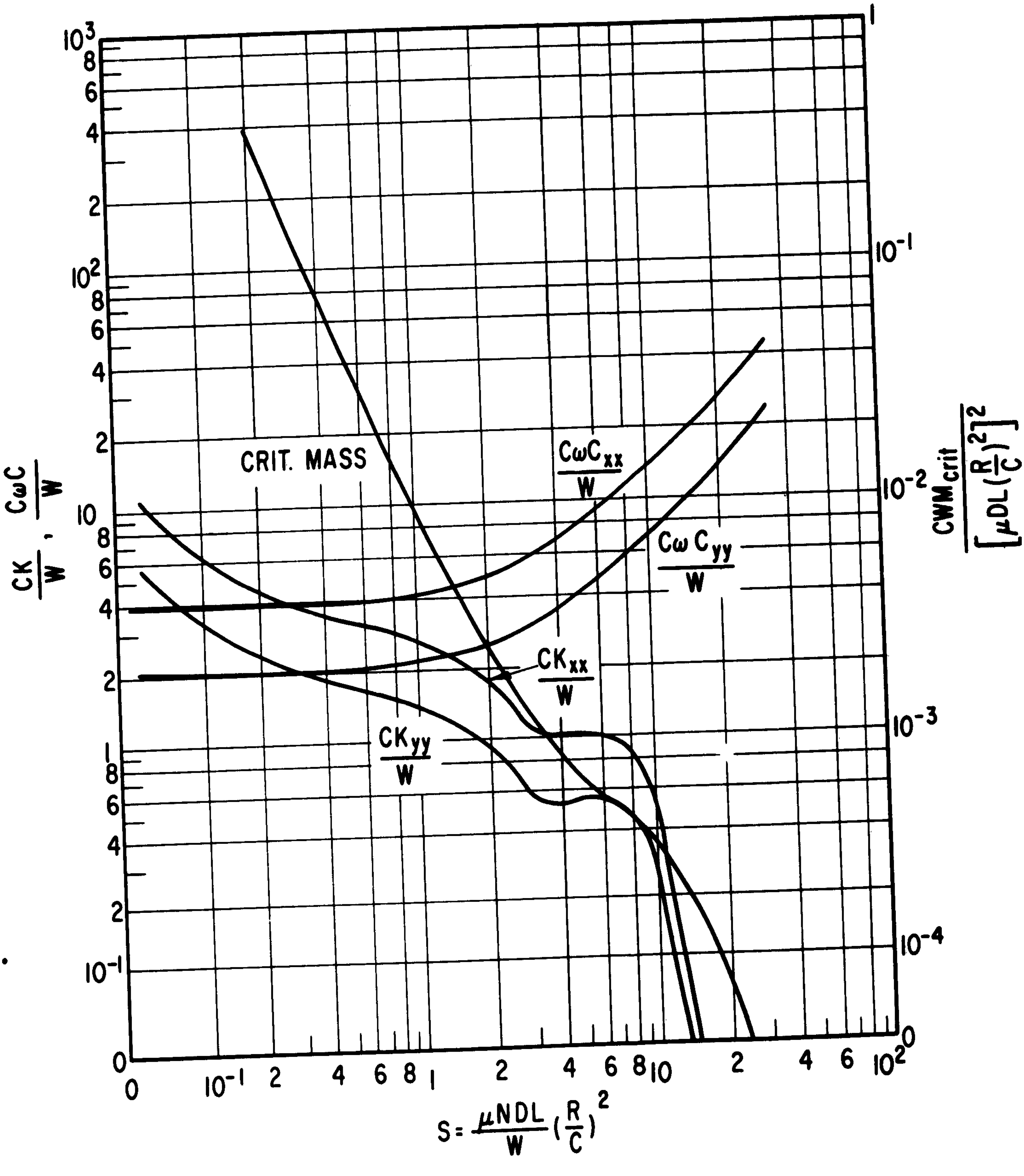


Fig. 5 Five 60° Tilting Pads, Centrally pivoted, L/D = .5, L/B = .955, C'/C = 1. Load between pads. No pad inertia.

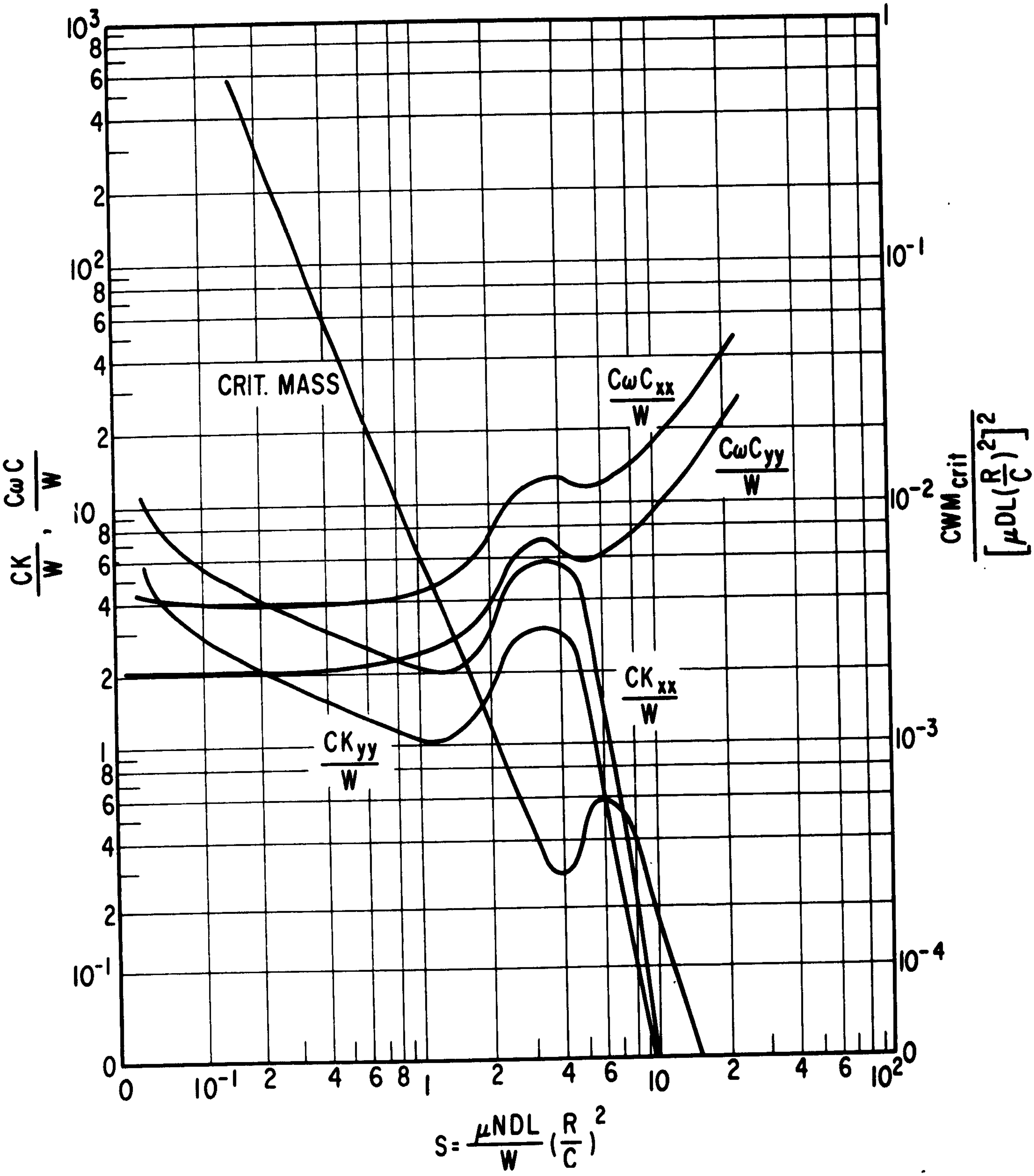


Fig. 6 Five 60° Tilting Pads, Centrally pivoted, L/D = .75, L/B = 1.432, C'/C = 1. Load between pads. No pad inertia.

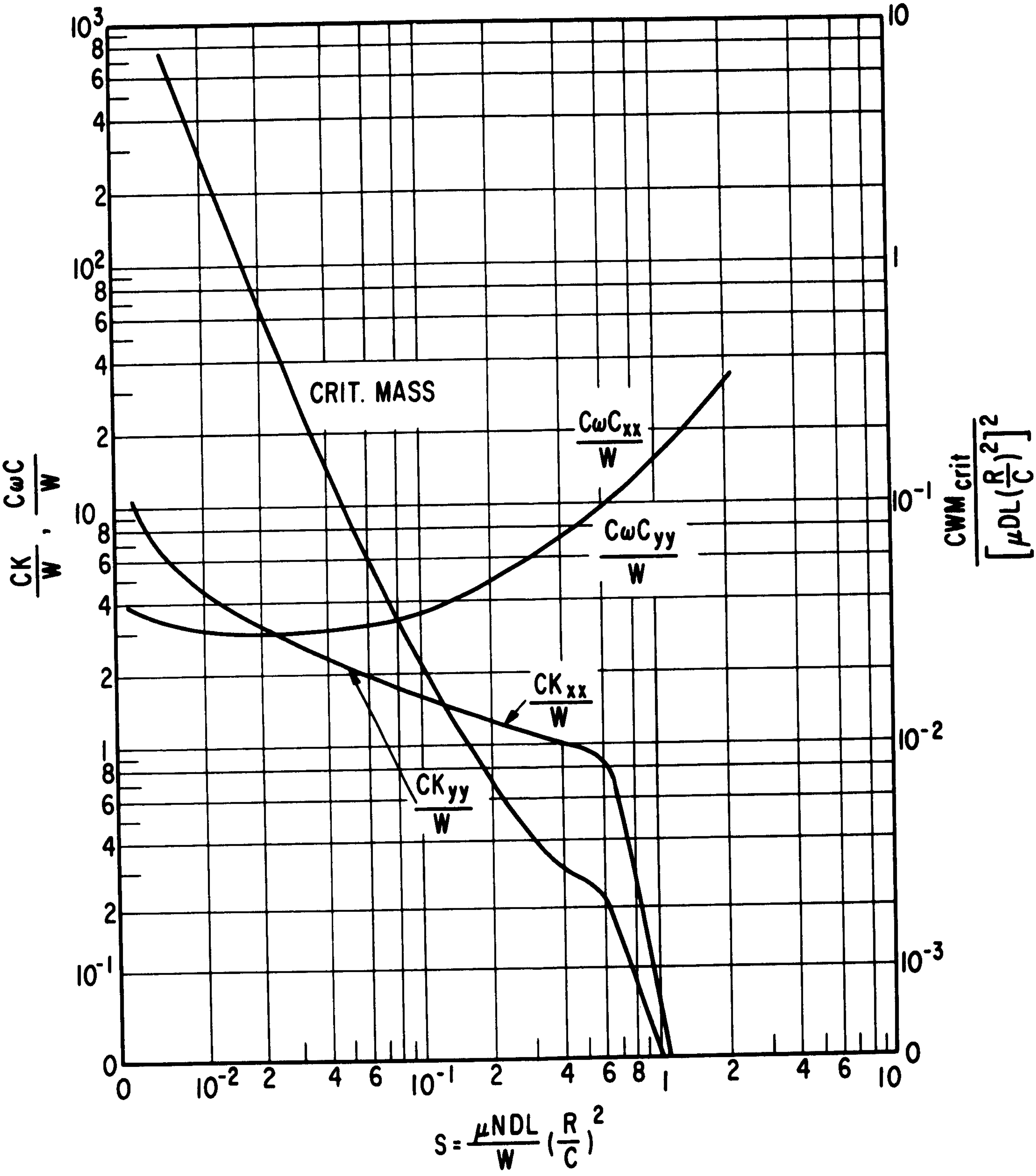


Fig. 7 Four 80° Tilting Pads, Centrally pivoted, L/D = .5, L/B = .716, C'/C = 1. Load between pads. No pad inertia.

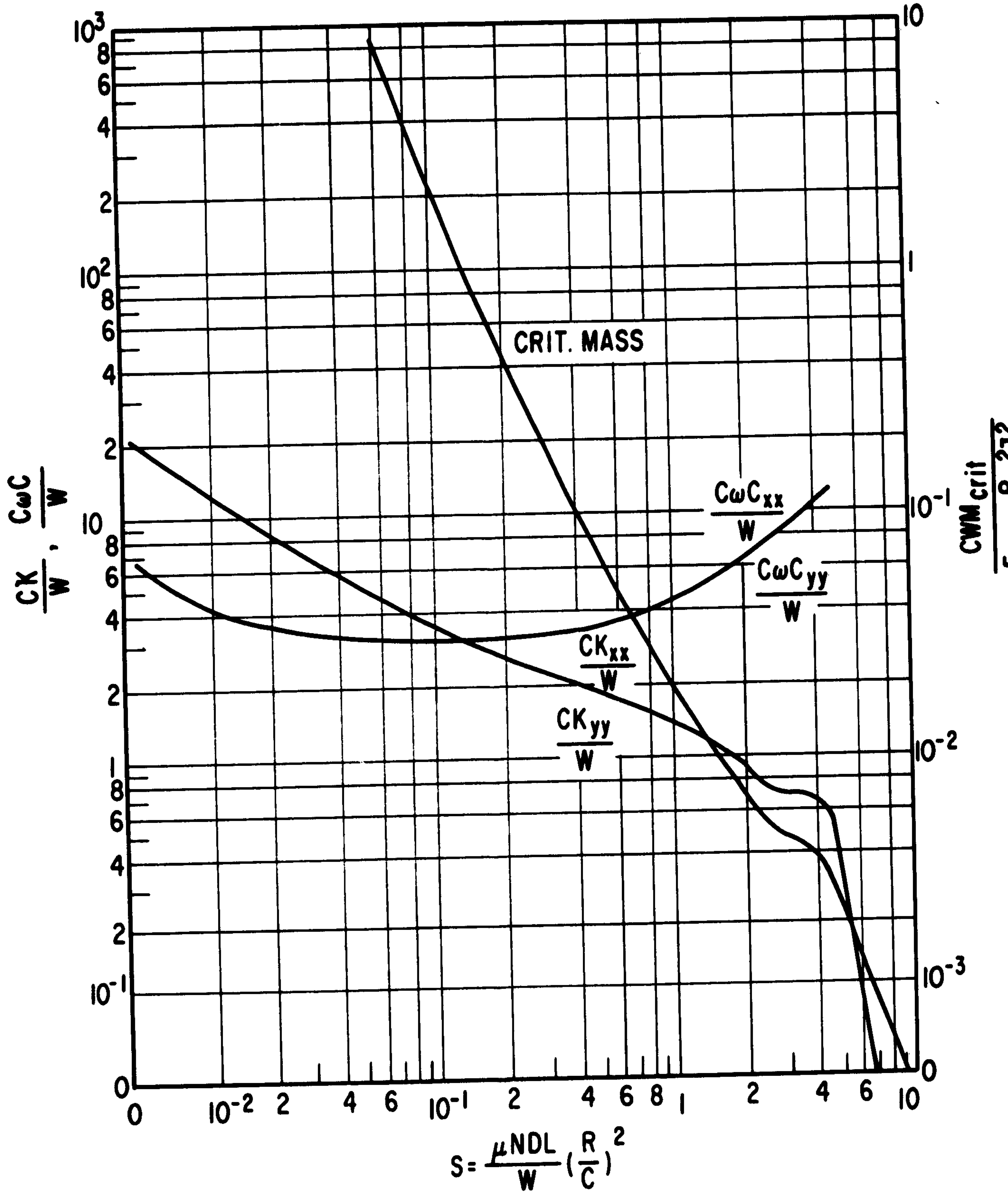


Fig. 8 Four 80° Tilting Pads, Centrally pivoted, L/D = .75, L/B = 1.074, C'/C = 1. Load between pads. No pad inertia.

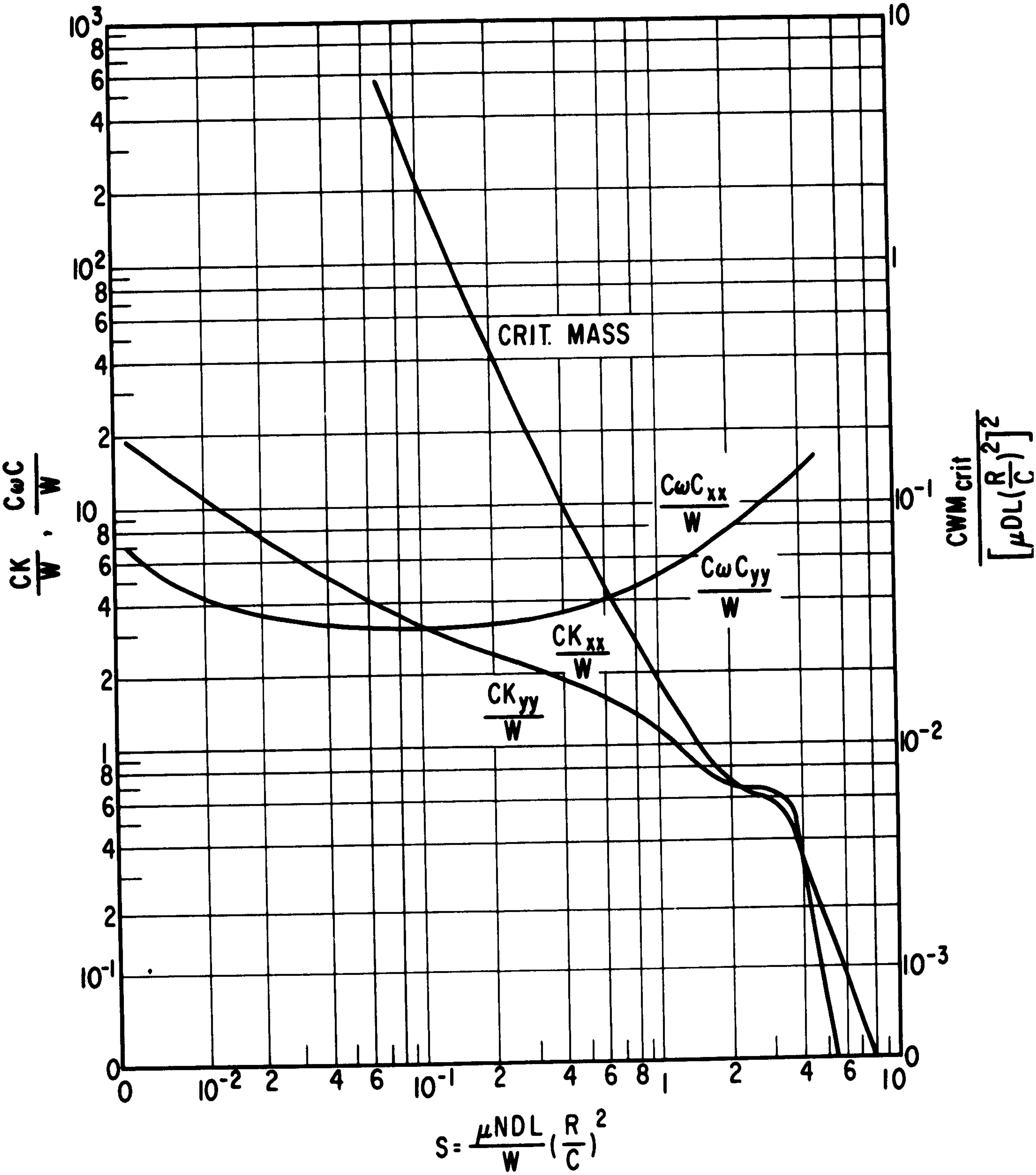


Fig. 9 Four 80° Tilting Pads, Centrally pivoted, L/D = 1.0, L/B = 1.432, C'/C = 1. Load between pads. No pad inertia.

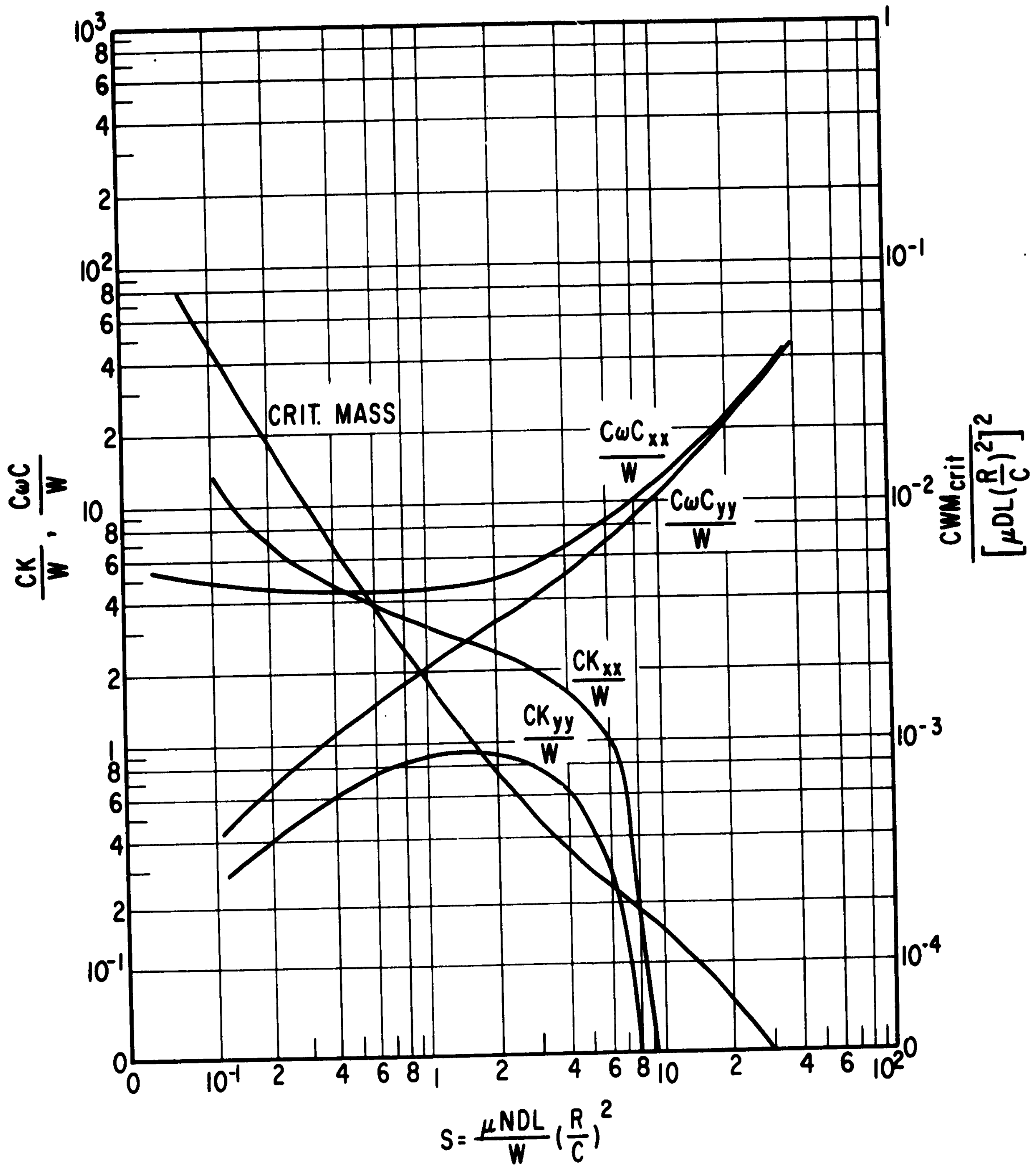


Fig. 10 Six 50° Tilting Pads, Centrally pivoted, L/D = .5, L/B = 1.146, C'/C = 1. Load on pad. No pad inertia.

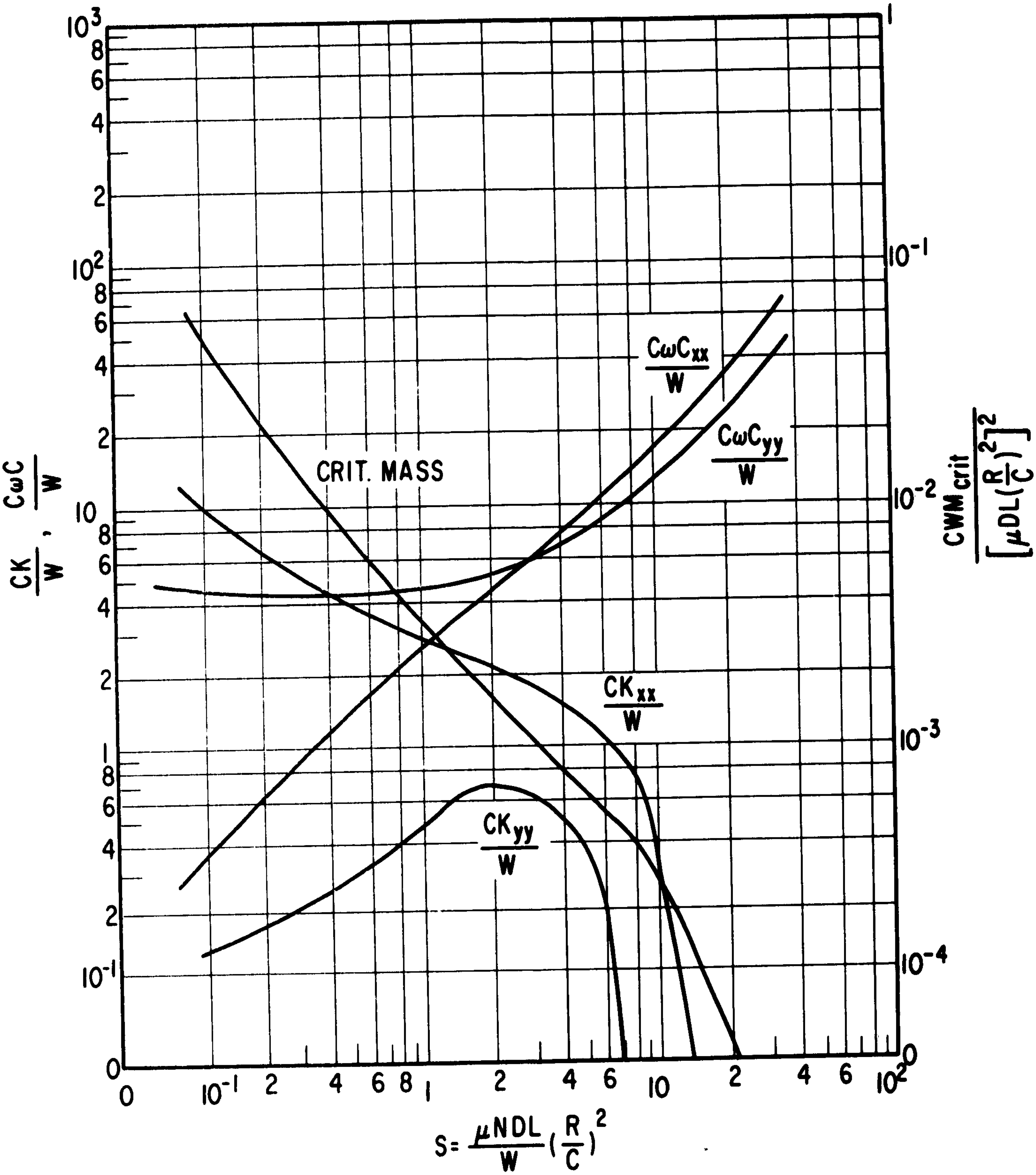


Fig. 11 Five 60° Tilting Pads, Centrally pivoted, L/D = .5, L/B = .955, C'/C = 1. Load on pad. No pad inertia.

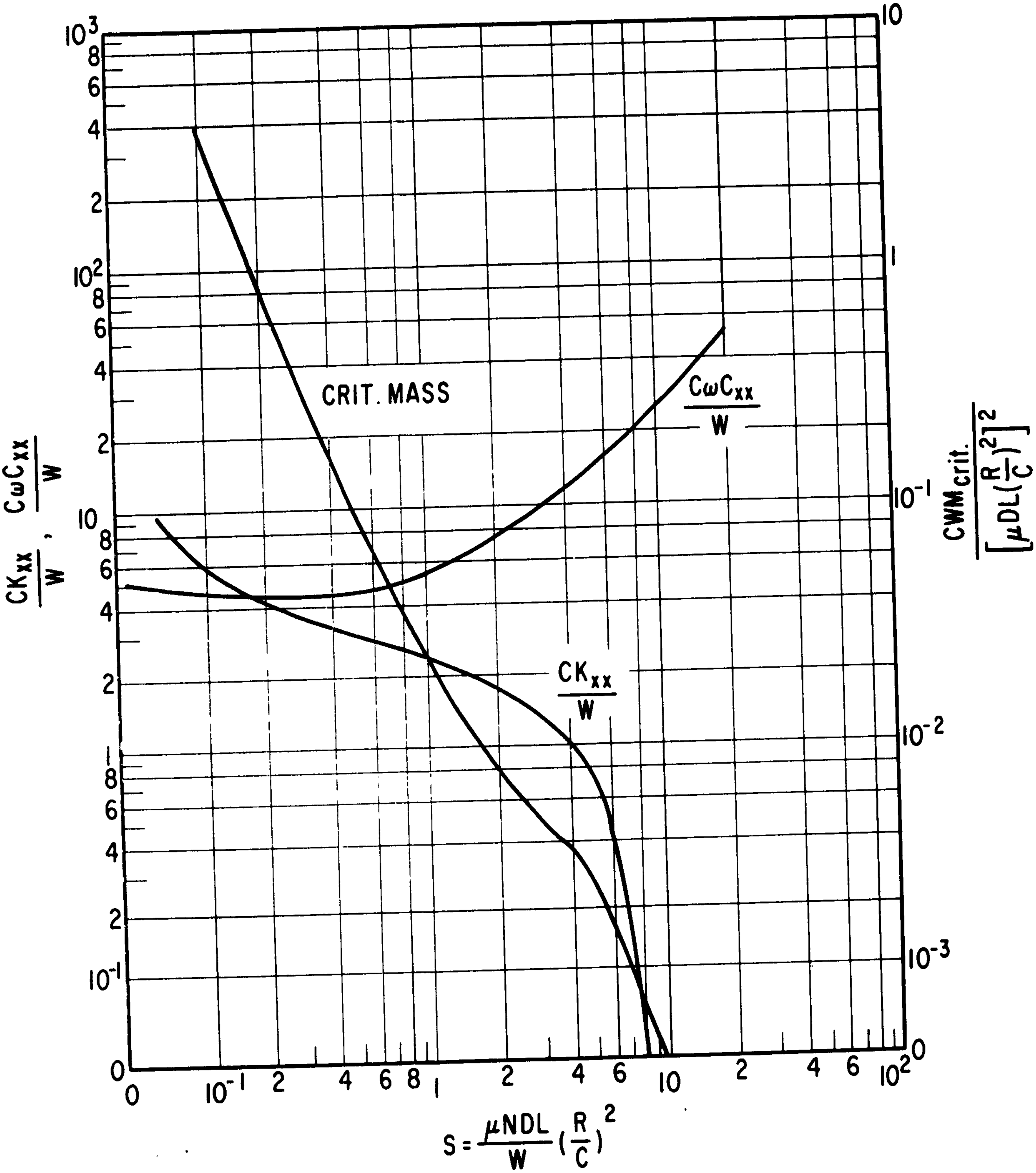


Fig. 12 Four 80° Tilting Pads, Centrally pivoted, L/D = .75, L/B = 1.074, C'/C = 1. Load on pad. No pad inertia.

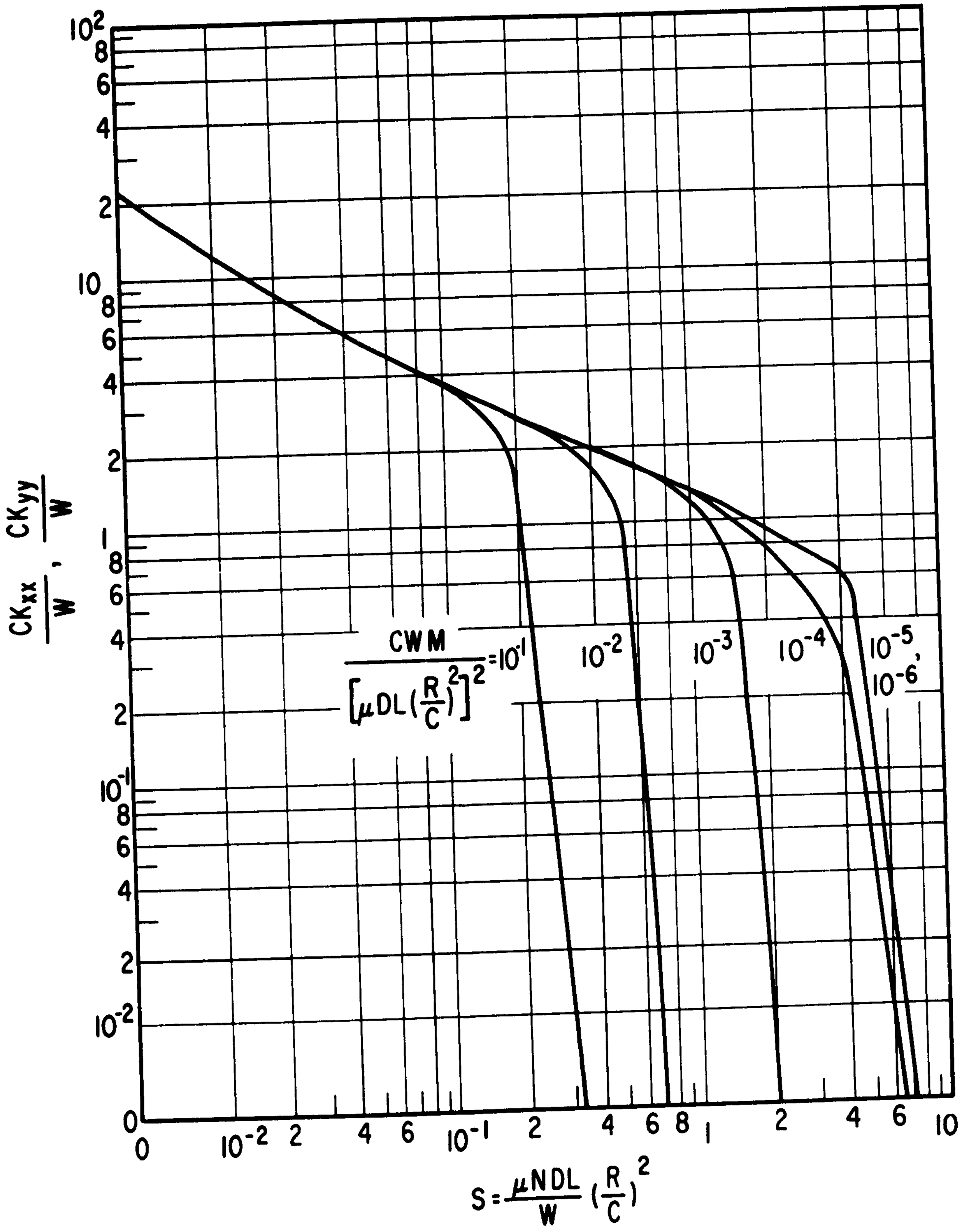


Fig. 13 Four 80° Tilting Pads, Centrally pivoted, L/D = .75, L/B = 1.074, C'/C = 1. Load between pads. Effect of pad inertia.

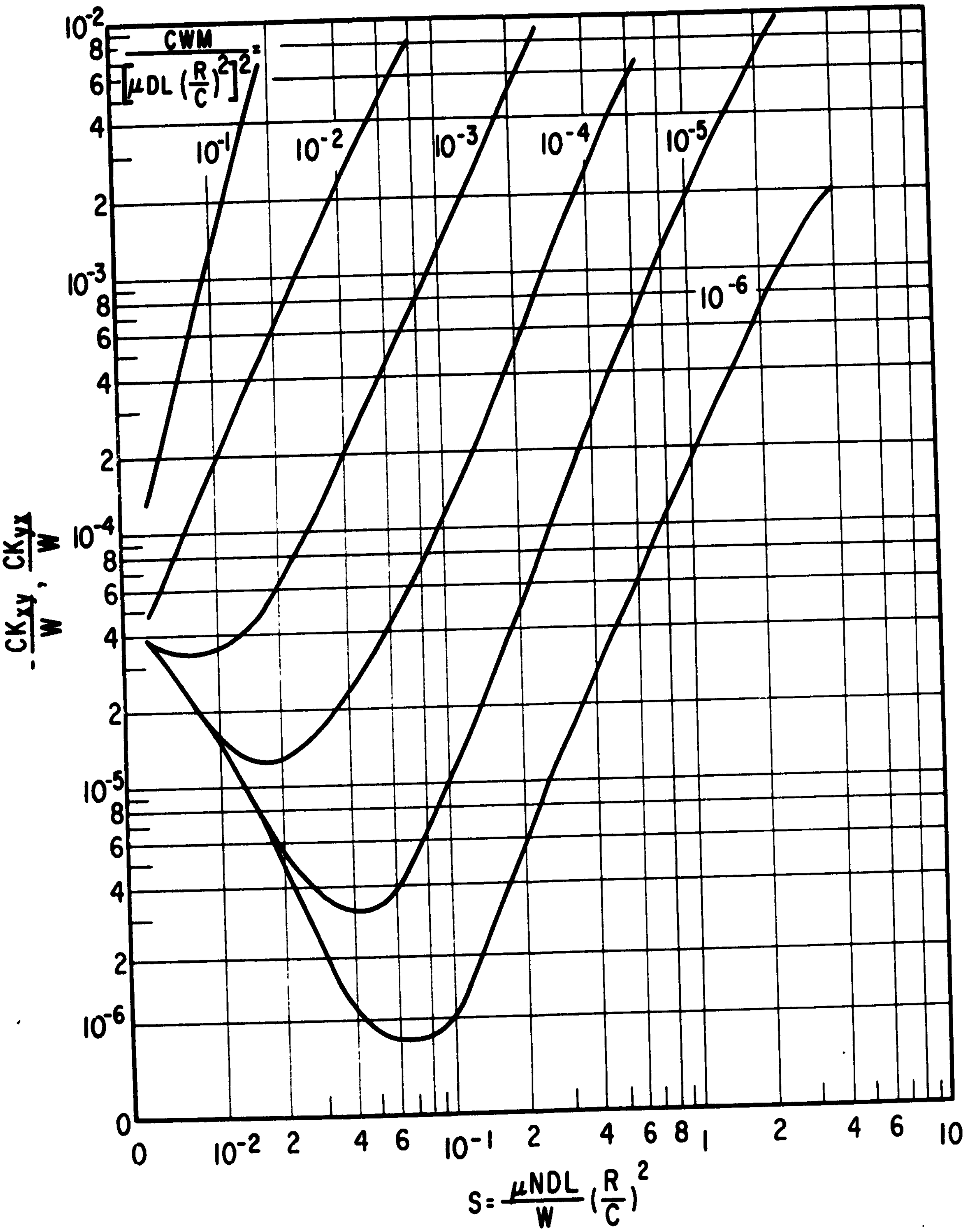


Fig. 14 Four 80° Tilting Pads, Centrally pivoted, L/D = .75, L/B = 1.074, C'/C = 1. Load between pads. Effect of pad inertia

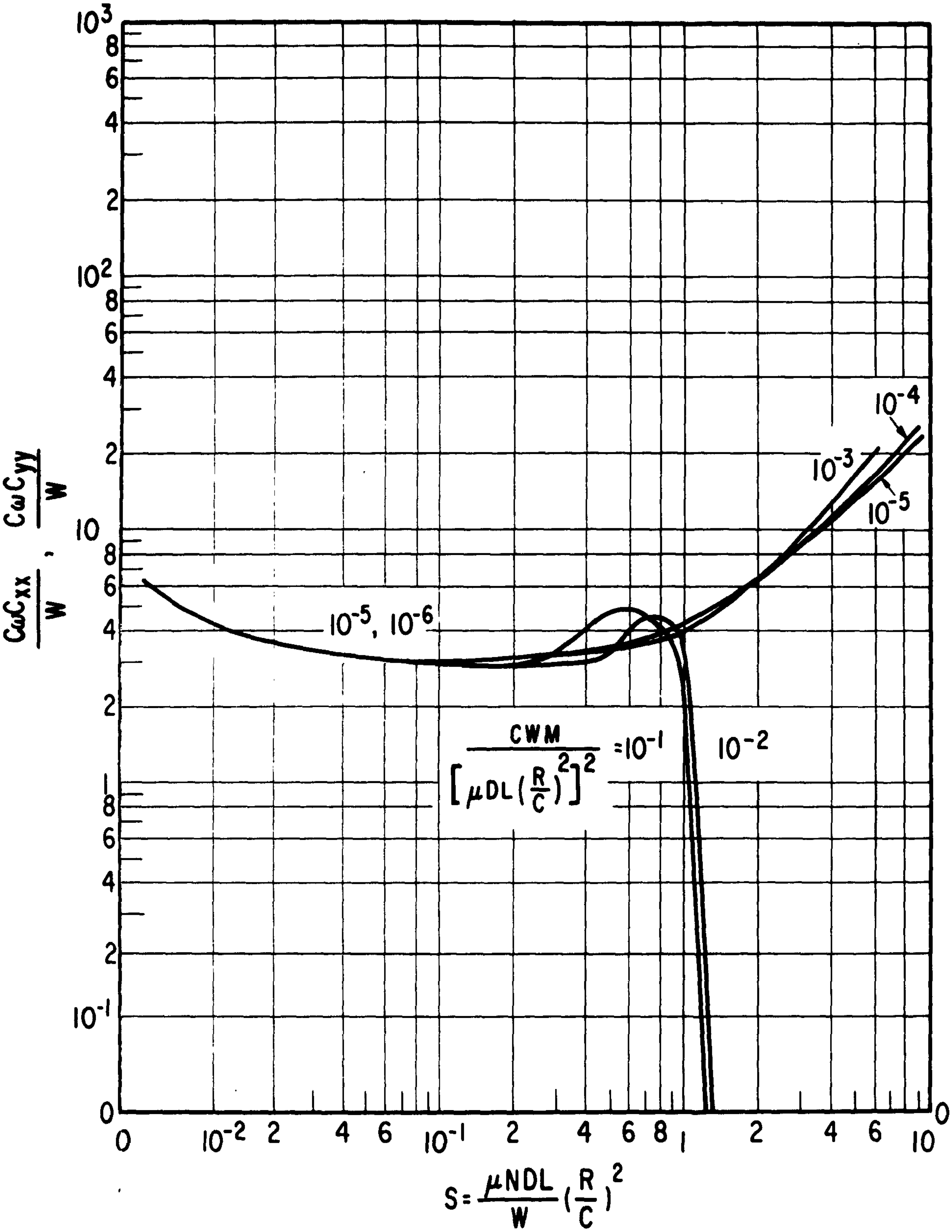


Fig. 15 Four 80° Tilting Pads, Centrally pivoted, L/D = .75, L/B = 1.074, C'/C = 1. Load between pads. Effect of pad inertia.

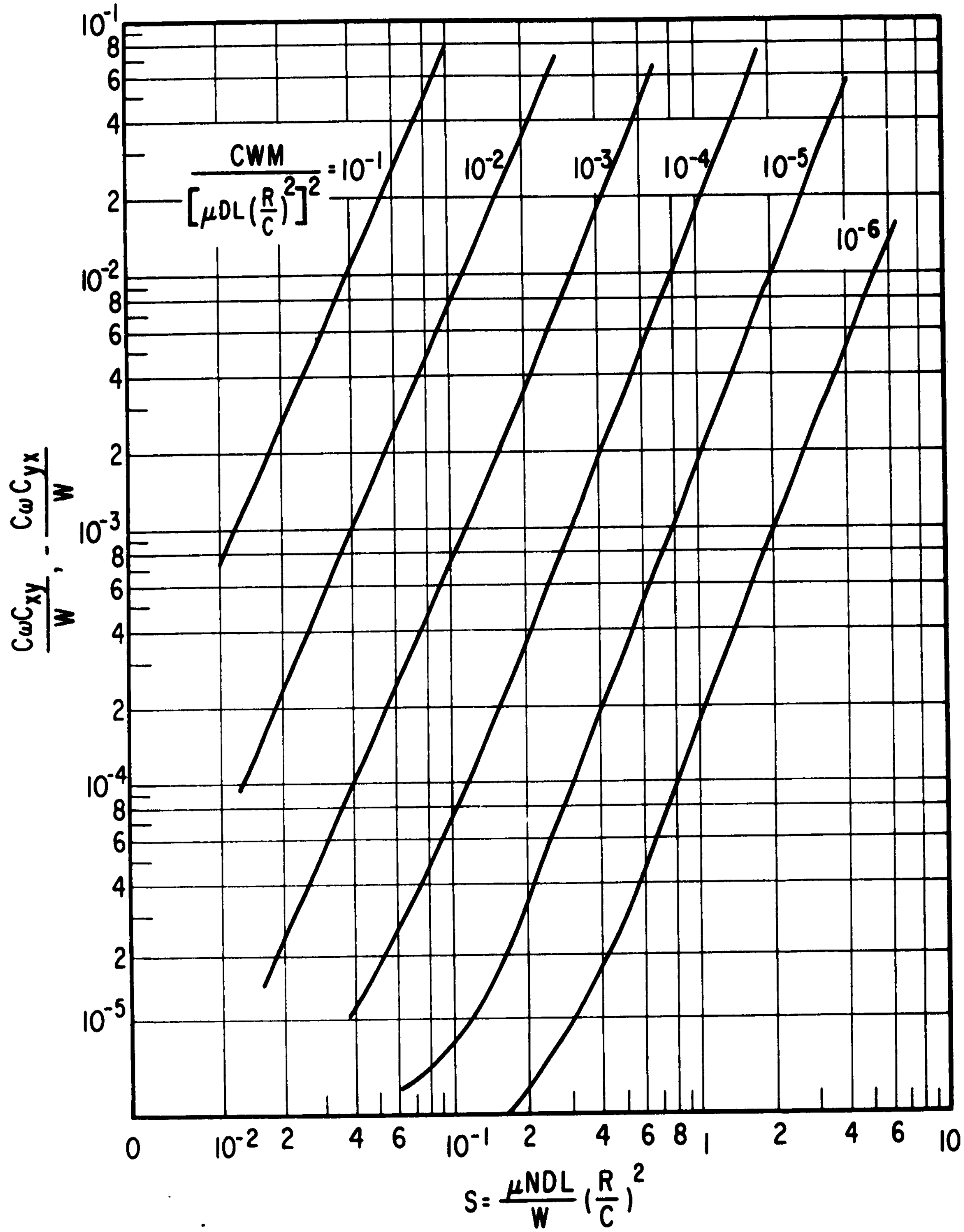


Fig. 16 Four 80° Tilting Pads, Centrally pivoted, L/D = .75, L/B = 1.074, C'/C = 1. Load between pads. Effect of pad inertia.

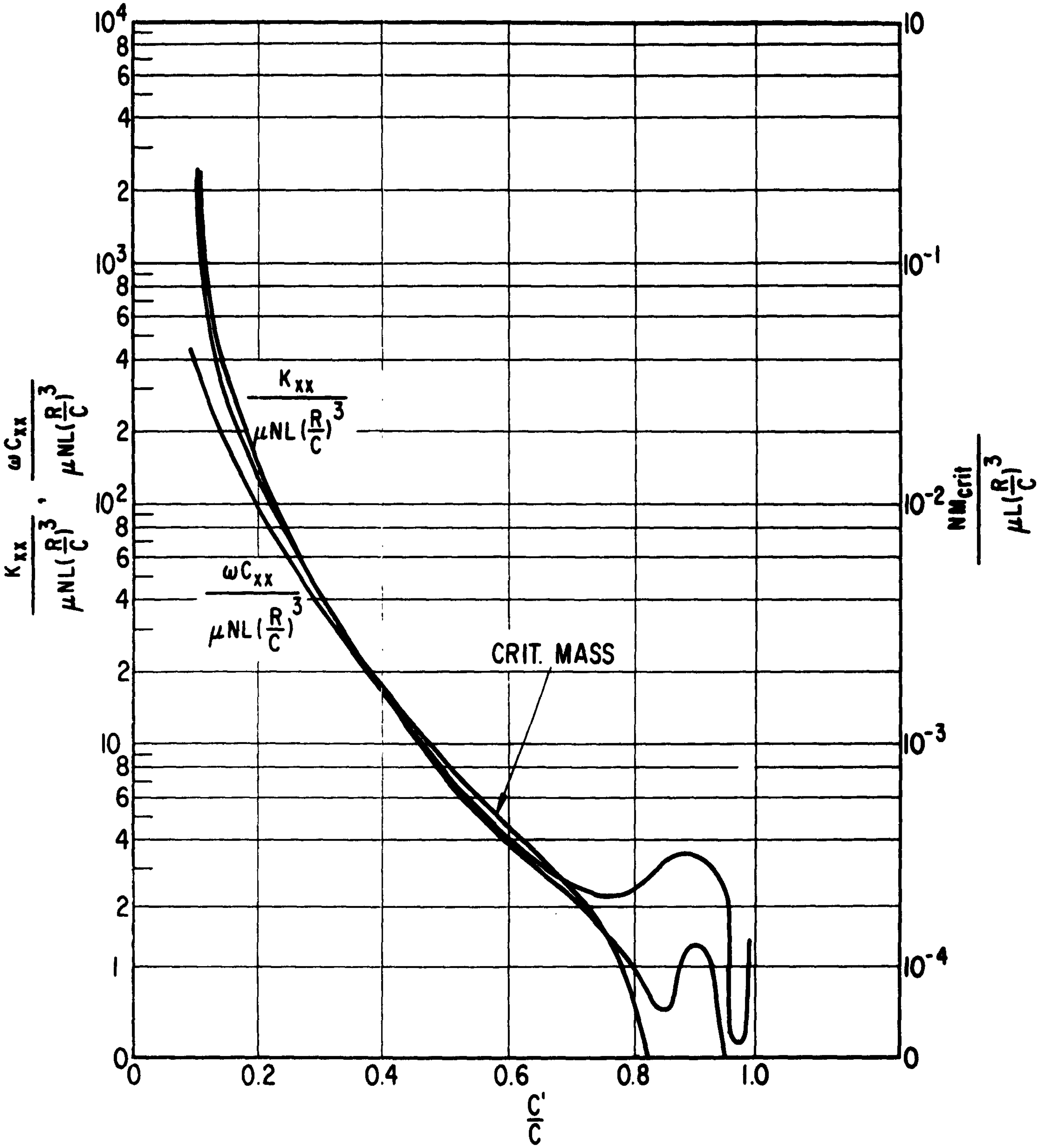


Fig. 17 Twelve 25° Tilting Pads, Centrally pivoted, L/D = .25, L/B = 1.146. Vertical rotor. Effect of pre-load (1-C'/C).

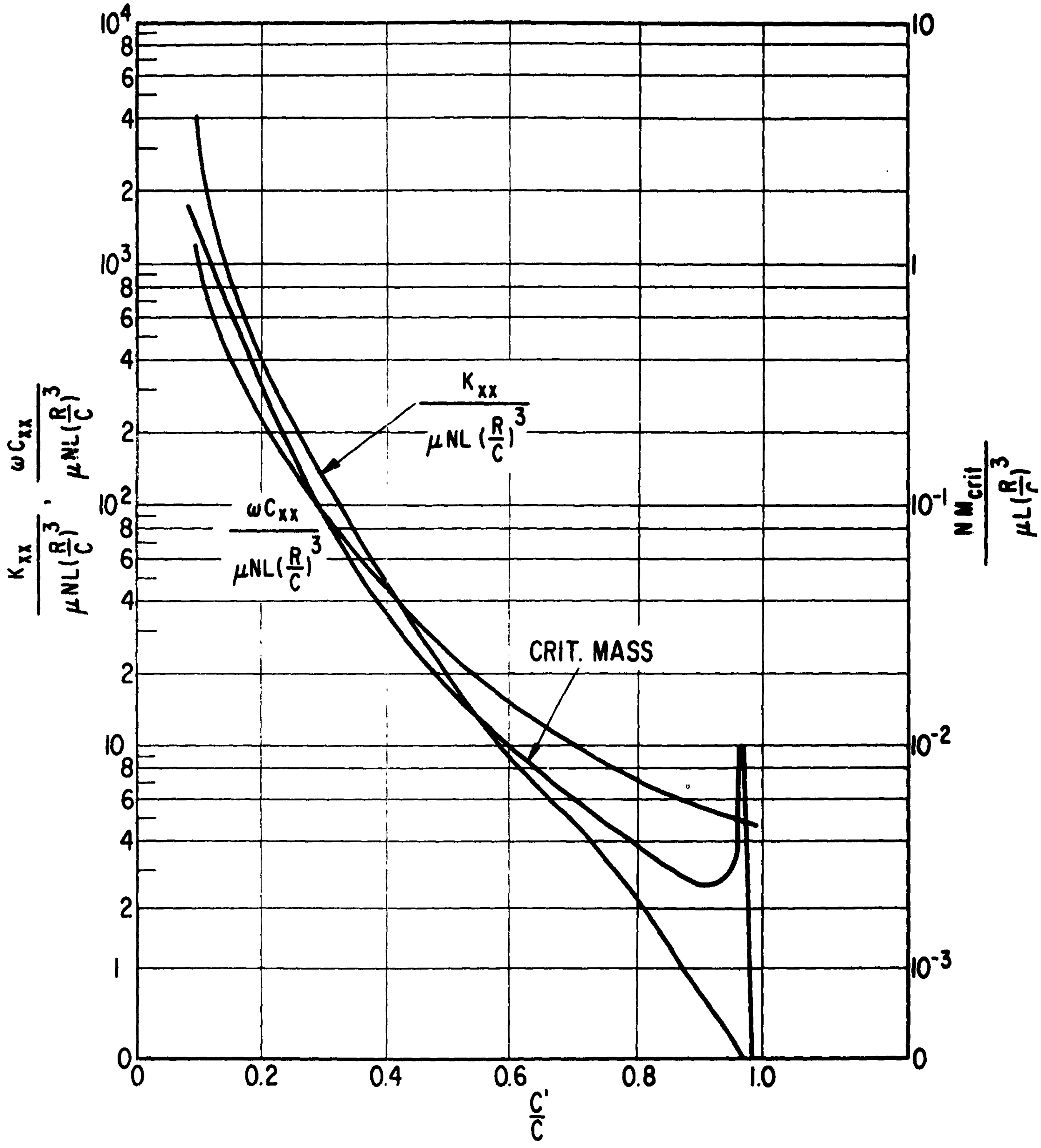


Fig. 18 Six 50° Tilting Pads, Centrally pivoted, L/D = .75, L/B = 1.146. Vertical Rotor. Effect of pre-load (1-C'/C).

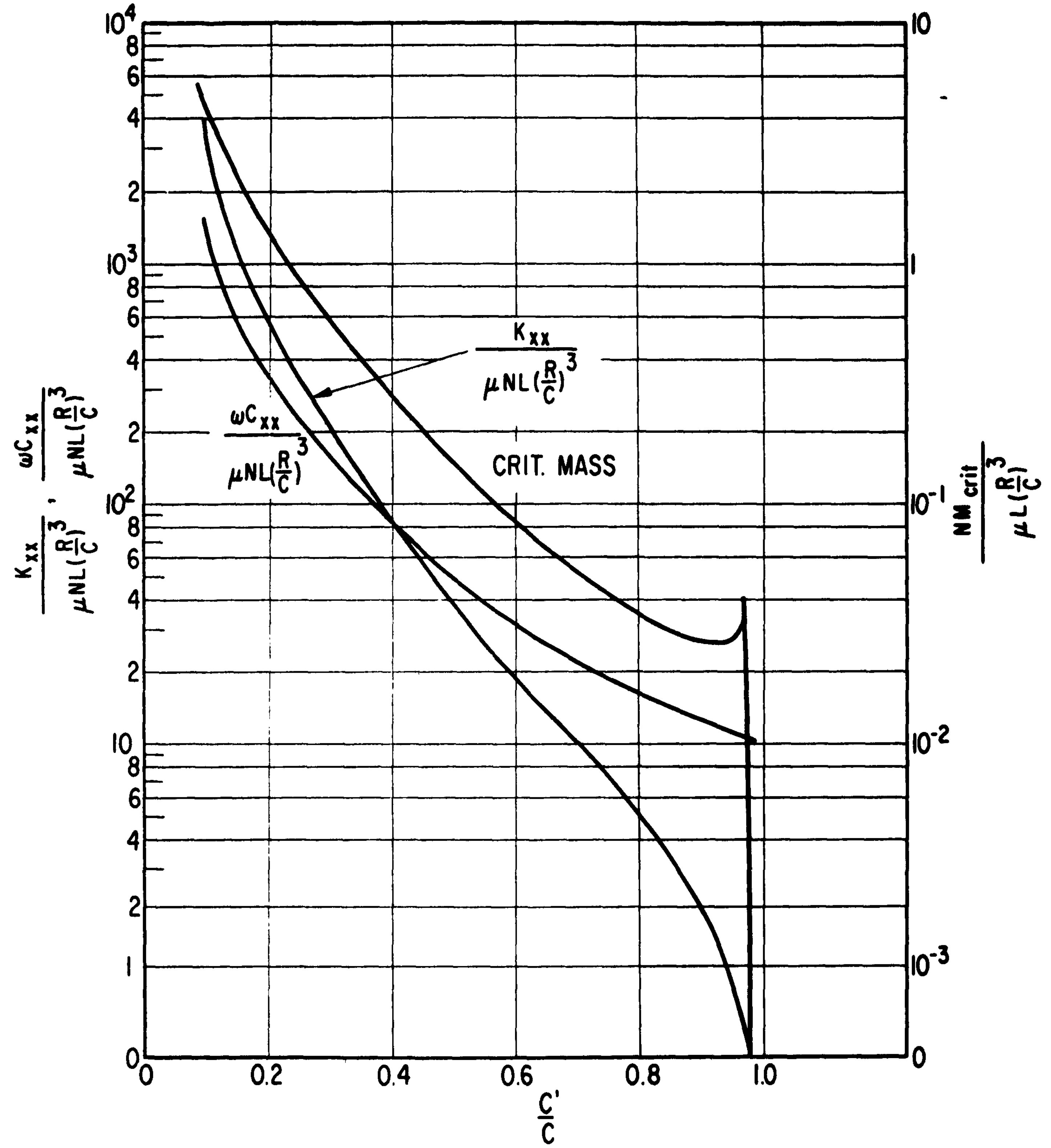


Fig. 19 Four 80° Tilting Pads, Centrally pivoted, L/D = .75, L/B = 1.074. Vertical Rotor. Effect of pre-load (1-C'/C).

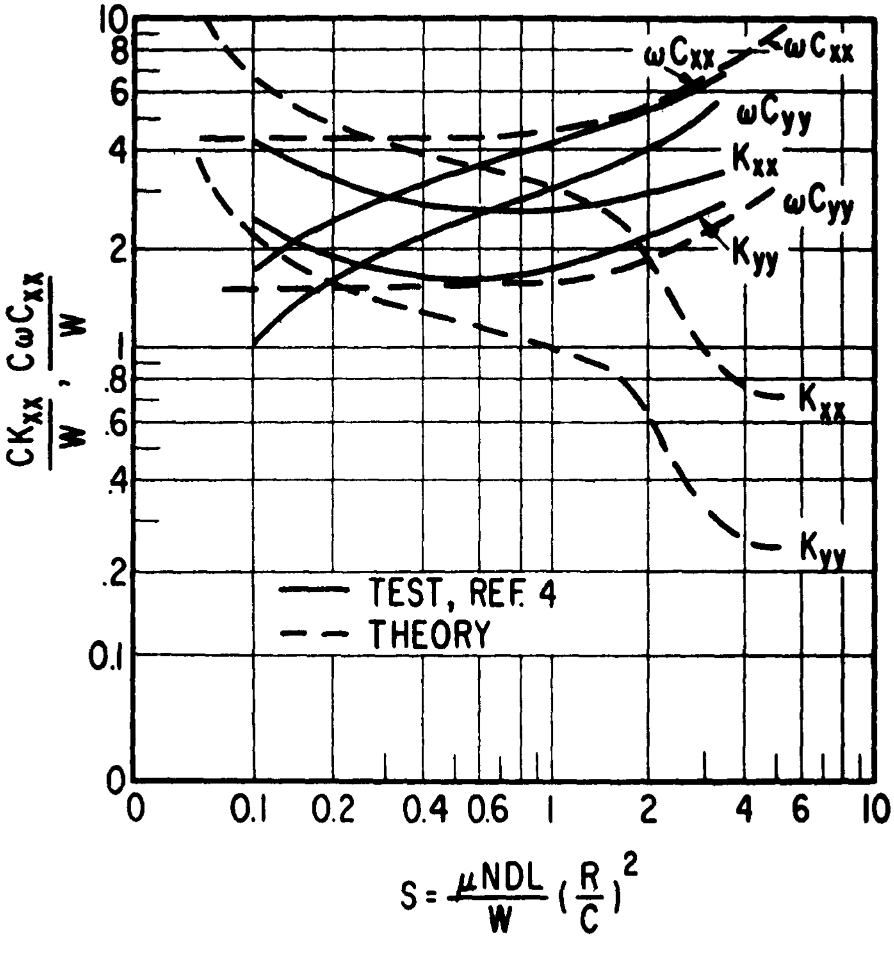


Fig. 20 Six 50° Tilting pads, Centrally pivoted, L/D = .75, L/B = 1.719, C'/C = 1. Load between pads. Comparison with Test Data.

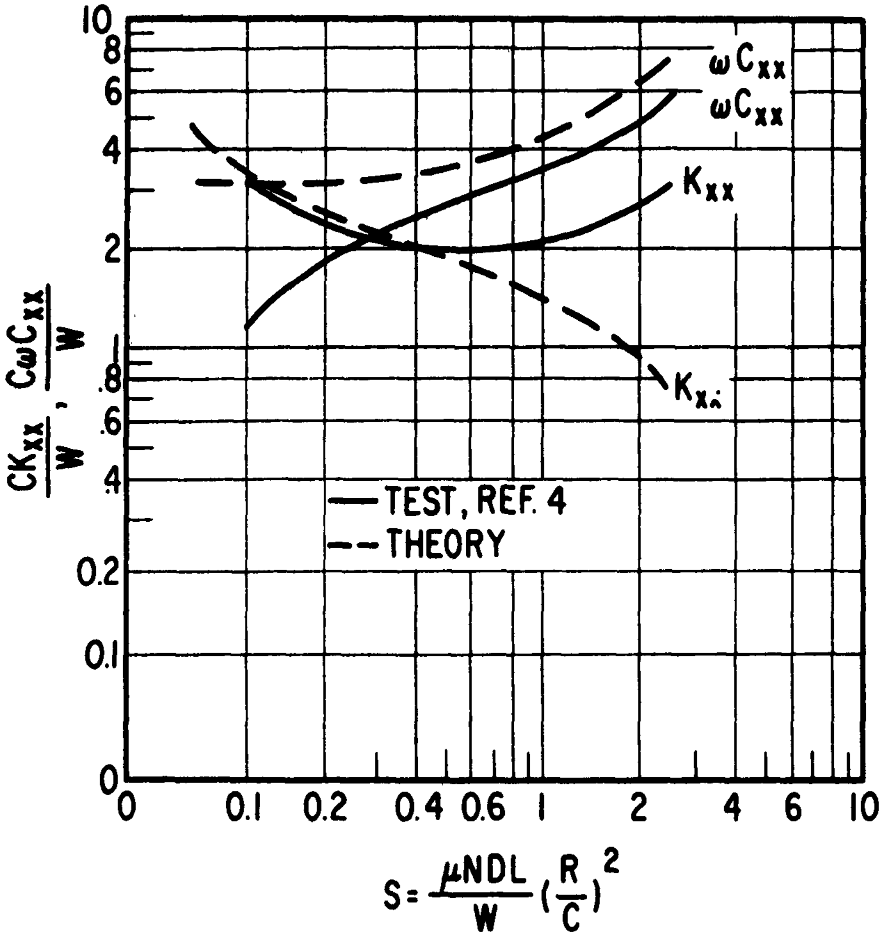


Fig. 21 Four 80° Tilting Pads, Centrally pivoted, L/D = .75, L/B = 1.074, C'/C = 1. Load between pads. Comparison with Test Data.

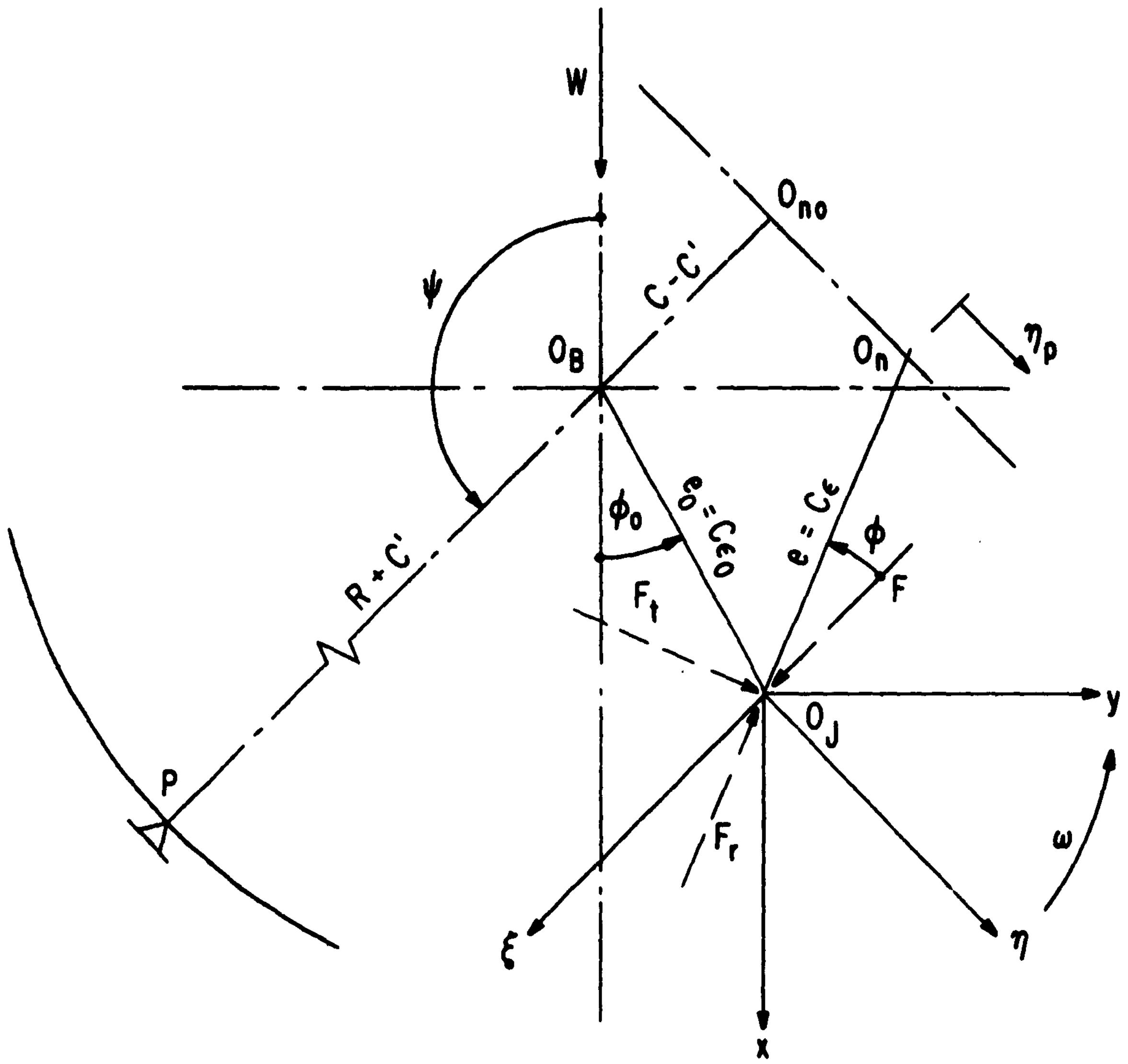


Fig. 22 Coordinate Systems for Analysis.

NOMENCLATURE

C	Pad clearance (radius of curvature of pad minus journal radius)inch.
C'	Pivot circle clearance (radius of pivot circle minus journal radius) inch
$C_{xx}, C_{xy}, C_{yx}, C_{yy}$	Bearing damping coefficients, lbs.sec/in.
$C_{\xi\xi}, C_{\xi\eta}, C_{\eta\xi}, C_{\eta\eta}$	Fixed pad damping coefficients, lbs.sec/in.
$C'_{\xi\xi}, C'_{\xi\eta}, C'_{\eta\xi}, C'_{\eta\eta}$	Tilting pad damping coefficients, lbs.sec/in.
D	Journal diameter, inch
e	Journal center eccentricity with respect to pad center, inch
e_o	Journal center eccentricity with respect to bearing center, inch
F	Load on pad. lbs.
F_r, F_t	Radial and tangential components of pad load, lbs.
F_ξ, F_η	Components in ξ and η-direction of pad load,lbs (see Fig. 22)
f	$= F/\lambda\omega$, dimensionless pad force
f_r, f_t	Radial and tangential components of dimensionless pad force
f_ξ, f_η	Components in ξ and η-direction of dimensionless pad force
I	Transverse mass moment of inertia of shoe around pivot, lbs.in.sec^2
$K_{xx}, K_{xy}, K_{yx}, K_{yy}$	Bearing spring coefficients, lbs/in.
$K_{\xi\xi}, K_{\xi\eta}, K_{\eta\xi}, K_{\eta\eta}$	Fixed pad spring coefficients, lbs/in.
$K'_{\xi\xi}, K'_{\xi\eta}, K'_{\eta\xi}, K'_{\eta\eta}$	Tilting pad spring coefficients, lbs/in.
L	Bearing length, inch
M	$= I/R^2_p$, equivalent pad mass, $\text{lbs.sec}^2/\text{in}$
M_{crit}	Value of equivalent pad mass to cause pad motion resonance, $\text{lbs.sec}^2/\text{in}$
N	Rotational speed of journal, RPS
p, q	Coefficients defined by Eq. (31) and (32)
R	Journal radius, inch

R_p	Radius from pad center to actual pivot point of pad, inch
S	$= (\mu NDL/W) \cdot (R/C)^2$, bearing Sommerfeld number
S_p	$= (\mu NDL/F) \cdot (R/C)^2$, pad Sommerfeld number
W	Bearing load, lbs.
x, y	Coordinates of journal center with respect to the bearing, See Fig. 22, inch
ε	$= e/C$, eccentricity ratio with respect to the pad center
ε_0	$= e_o/C$, eccentricity ratio with respect to the bearing center
η_p	Amplitude for pad center motion, see Fig. 22, inch
η_0	Amplitude of the center of a massless pad, inch.
λ	$= (\mu RL/\pi) \cdot (R/C)^2$ bearing coefficient
μ	Lubricant viscosity, lbs.sec./in^2
ξ, η	Coordinates of journal center with respect to the pad, see Fig.22 inch
φ	Attitude angle with respect to the pad load line, radians
φ_0	Attitude angle with respect to the bearing load line, radians
ψ	Angle from vertical (negative x-axis) to pad pivot point, see Fig. 22 inch
ω	Angular speed of shaft, radian/sec.